Salahaddin University-Erbil **College of Science** **Department of Mathematics** **3rd Stage**		**Subject: Mechanical Analysis** **Period: 60 Minutes** **Date: 28 April 2022** **First Examination** **Second Semester 2021-2022**

Q.1/ Is the force field $\vec{F} = -(2ax + by)\hat{\imath} - (bx + 2cy)\hat{\jmath}$ conservative? If it is conservative find the corresponding potential energy function.

[12 Marks]

Q.2/ Write the differential equation of the following: *[12 Marks]*

- **i)** Differential equation for constrained motion of a particle.
- **ii)** Differential equation for projectile motion with quadratic air resistance.
- **iii)** Differential equation for motion of the charged particle in electromagnetic field.
- **iv)** Differential equation for motion of the particle on the curve.

Q.3/ Consider a particle sliding under gravity in a smooth cycloidal trough, represented by the parametric equation:

$x = A(2\theta + \sin 2\theta)$ where A is constant and θ is the parameter.

Show that the particle undergoes simple harmonic motion. *[12 Marks]*

Good Luck *Asst. Prof. Dr. Tahseen G. Abdullah*

Ans. of Q.1:

$$\vec{F} = -(2ax + by)\hat{i} - (bx + 2cy)\hat{j}$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -(2ax + by) & -(bx + 2cy) & 0 \end{vmatrix} = \hat{i}(0+0) + \hat{j}(0-0) + \hat{k}(-b+b) = 0$$

The force is conservative and there is a potential energy function such that: $\vec{F} = -\vec{\nabla}V$

$$F_x = -\frac{\partial V}{\partial x} \Rightarrow V(y,z) = -\int F_x dx = \int (2ax + by)dx = ax^2 + bxy + c1$$

$$F_y = -\frac{\partial V}{\partial y} \Rightarrow V(x,z) = -\int F_y dy = \int (bx + 2cy)dy = bxy + cy^2 + c2$$

$$F_z = -\frac{\partial V}{\partial z} \Rightarrow V(x,y) = -\int F_z dz = 0 + c3$$

Thus, the potential energy is the combination of the potentials:

$$V(x,y,z) = ax^2 + bxy + cy^2 + c$$

Ans. of Q.2:

i) Differential equation for constrained motion of a particle.

$$m\vec{a} = \vec{F} + \vec{R}$$

ii) Differential equation for projectile motion with quadratic air resistance.

$$m\ddot{\vec{\mathbf{r}}} = m\vec{\mathbf{g}} - cv^2\hat{\tau} \qquad m\ddot{\vec{\mathbf{r}}} = m\vec{\mathbf{g}} - cv\vec{v}$$

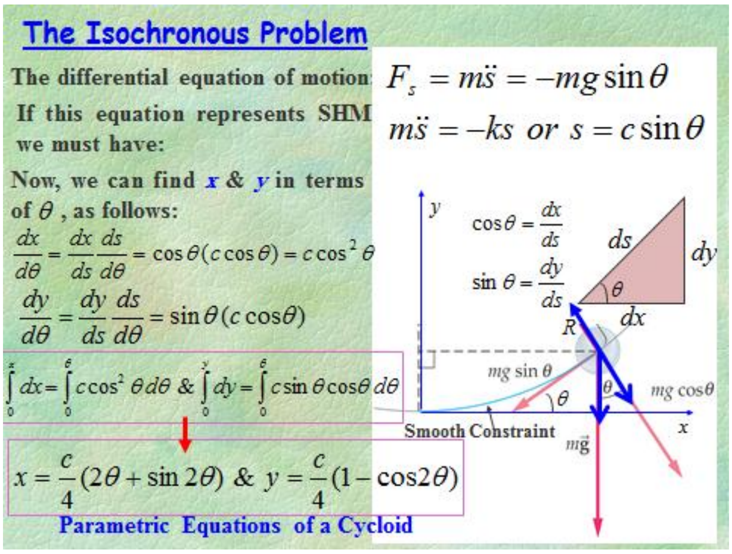
iii) Differential equation for motion of the charged particle in electromagnetic field.

$$\vec{F} = m\vec{a} = m\frac{d\vec{\mathbf{v}}}{dt} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

iv) Differential equation for motion of the particle on the curve.

$$F_s = m\ddot{s} = -\frac{dV(s)}{ds}$$

Ans. of Q.3:



$$x = A(2\theta + sin2\theta)$$

$$\frac{dx}{d\theta} = \frac{dx}{ds}\frac{ds}{d\theta} \longrightarrow 2A + 2A\cos2\theta = cos\theta\frac{ds}{d\theta}$$

$$2A(1 + \cos2\theta) = cos\theta\frac{ds}{d\theta} \longrightarrow 4A\cos^2\theta = cos\theta\frac{ds}{d\theta}$$

$$\frac{ds}{d\theta} = 4A\cos\theta \longrightarrow \int_0^s ds = \int_0^\theta 4A\cos\theta d\theta$$

$s = 4Asin\theta = csin\theta$ **where c=4*A***