Salahaddin University-Erbil **College of Science** **Department of Math** **3rd Stage** **Secnd Semester 2021-2022**		**Subject: Mechanical Analysis** **Period: 30 Minutes** **Date: May 2022** **Quiz Exam**

Q.1/ Find the kinetic energy for motion of the individual particles relative to the center of mass for the following system of 3-particles: *[10 Marks]*

$$\vec{v}_1 = 3\hat{i} \quad , \quad \vec{v}_2 = \hat{i} + 2\hat{j} \;\; \& \;\; \vec{v}_3 = 2\hat{i} + \hat{j} + 3\hat{k} \qquad m_1 = m_2 = m_3 = 1$$

Good Luck *Asst. Prof. Dr. Tahseen G. Abdullah*

Ans. of Q.:

$$T = \tfrac{1}{2} m v_{cm}^2 + \sum_{i=1}^{n} \tfrac{1}{2} m_i v_i'^2 \qquad m = \sum m_i = 1+1+1 = 3$$

$$\vec{v}_{cm} = \frac{1}{m} \sum m_i \vec{v}_i$$

$$\vec{v}_{cm} = \frac{1}{3}(\vec{v}_1 + \vec{v}_2 + \vec{v}_3) = \frac{1}{3}(3\hat{i} + \hat{i} + 2\hat{j} + 2\hat{i} + \hat{j} + 3\hat{k}) = \frac{6}{3}\hat{i} + \frac{3}{3}\hat{j} + \frac{3}{3}\hat{k} = 2\hat{i} + \hat{j} + \hat{k}$$

$$T_{cn} = \frac{1}{2} m v_{cm}^2 = \frac{1}{2} \times 3 \times (4+1+1) = 9$$

$$T = \sum_i \tfrac{1}{2} m_i v_i^2 = \tfrac{1}{2}(v_1^2 + v_2^2 + v_3^2) = \tfrac{1}{2}(9+5+4+1+9) = 14$$

$$\boxed{T' = T - T_{cn} = 14 - 9 = 5}$$

2nd way:

$$T' = \sum_i \tfrac{1}{2} m_i v_i'^2 = \tfrac{1}{2}(v_1'^2 + v_2'^2 + v_3'^2) \qquad \boxed{\vec{v}_i = \vec{v}_{cm} + \vec{v}_i'}$$

$$\vec{v}_1' = \vec{v}_1 - \vec{v}_{cm} = 3\hat{i} - (2\hat{i} + \hat{j} + \hat{k}) = \hat{i} - \hat{j} - \hat{k} \quad \Rightarrow v_1'^2 = 1+1+1 = 3$$

$$\vec{v}_2' = \vec{v}_2 - \vec{v}_{cm} = \hat{i} + 2\hat{j} - (2\hat{i} + \hat{j} + \hat{k}) = -\hat{i} + \hat{j} - \hat{k} \quad \Rightarrow v_2'^2 = 1+1+1 = 3$$

$$\vec{v}_3' = \vec{v}_3 - \vec{v}_{cm} = (2\hat{i} + \hat{j} + 3\hat{k}) - (2\hat{i} + \hat{j} + \hat{k}) = 2\hat{k} \quad \Rightarrow v_3'^2 = 4$$

$$T' = \tfrac{1}{2}(v_1'^2 + v_2'^2 + v_3'^2) = \tfrac{1}{2}(3+3+4) = \tfrac{1}{2}(10) = 5$$