Ministry of Higher Education & Scientific Research

Salahaddin University-Erbil

College of Science

Mathematical Department

3rd Year Stage

Academic Year 2023-2024

- *Course Book:*

Analytical Mechanics

- *Language of instruction:*
  The course might be given in English

- *Lecture's Name:*
  Asst. Prof. Dr. Tahseen G. Abdullah

Course Book

1. Course name	**Mechanical Analysis I & II**
2. Lecturer in charge	**Asst. Prof. Dr. Tahseen G. Abdullah**
3. Department/ College	**Physics, Science**
4. Contact	**e-mail: tahsee.abdullah@su.edu.krd** **Tel: 009647504496290**
5. Time (in hours) per week	**Theory: 3** **Practical: 0**
6. Office hours	**4**
7. Course code	
8. Teacher's academic profile	**I have more than 20 year experience. Teaching Analytical Mechanics for the last of 15 years. I taught also most of the physics subjects during my stay in the university including MSc and PhD courses. Regarding scientific activities, I also published about 10 scientific articles in national and international journals. Participation in different conferences and meeting over the world. I worked in many filed in the university. Now, I am a deputy dean of the college of science.** **BSc of Physics (1992)** **MSc of Solid State Physics (1994)** **PhD of Quantum Solid State(Computational) Physics (2008)** **Assist Lecturer -**1994 **Instructor** -2001 **Assist Prof.** -2016
9. Keywords	**Analytical Mechanics, Mechanics of Newton, Lagrangian and Hamiltonian Mechanics**

10. Course overview:

Theory/Concepts: The student will be able to recall basic principles of physics (velocity, acceleration, force, energy, and momentum) and use them to develop appropriate models to understand motion in three dimensions, noninertial reference systems, central forces and introductory Legrangian mechanics. In addition he/she will apply these principles by solving a wide range of physics problems using the appropriate mathematical techniques. Analysis: The student will be able to analyze physical situations related to kinematics, Newton's laws, oscillators, conservation laws and gravity and apply the techniques of vector analysis and differential equations to solve for motion in three dimensions, determine the effect that a noninertial system has on measurement, determine relationships between different physical quantities for orbital motion, and generate the equations of motion from Legranges equations.

Communication: The student will express physical theories and models with clarity and precision as they are related to phenomena existent in physics, biology, chemistry and everyday life Community: The student will be able to express how physics interfaces with decisions affecting society and value the importance of thinking through problems using appropriate problem solving techniques.

11. Course objective:

The course will give basic knowledge about the principles, the conceptions and methods in analytical mechanics based on Newton's, Langrange's and Hamilton's formulation of the laws of the classical mechanics. Also, provide a basis for further studies in classical mechanics and quantum mechanics. The student will be able to recall basic principles of physics (velocity, acceleration, force, energy, and momentum) and use them to develop appropriate models to understand motion in three dimensions, no inertial reference systems, central forces, systems of particles, collisions, rigid-body mechanics and Lagrangian and Hamiltonian mechanics. In addition he/she will apply these principles by solving a wide range of physics problems using the appropriate mathematical techniques. Some of the course topics are:

- Motion for forces with the following behaviour:
  - Fconst, F(t), F(x), F(v), F(x,v), F(x,v,t)
- Javelins, baseballs and golf balls (projectiles)
- Clocks (pendulums)
- Riding in a car (springs and shock absorbers)
- Lagrangian formulation
- Music of the spheres (gravity and orbits)
- Interactions and rockets
  Bicycles and Hurricanes (gyroscopes and coriolis)

12. Student's obligation

Assignments play an important role in keeping track of comprehending all the skills and problem-solving for topics. There will be weekly assignment sets that will be collected and graded. If you have difficulties with the questions, please come see me during my office hours or make an individual appointment. I prefer that you attempt the assignments on your own before seeking help. You are allowed to work with others on the assignments. However, the work must be your own. Do not simply copy what someone else has done. Also, if you are working in a group does not arrive at a common answer and copy it point-by-point for each group member! These assignments will be your primary means of learning the material, so please take them seriously!. Also, attendance is one of the mandatory tasks for students, since the key idea behind each topic the subject would be given during the lectures and labs. Finally, Exams are the true criterion for measuring the depth of student understands to the given material.

13. Forms of teaching

Different forms of teaching will be used to reach the objectives of the course: power point presentation for the head titles and definitions and summary of conclusions, classification of materials and any other illustrations, besides worksheet will be designed to let the chance for practicing on several aspects of the course in the classroom and laboratory. There will be classroom discussions and the lecture will give enough background to translate, solve, analyze, and evaluate problems sets, and different issues discussed throughout the course. To get the best of the course, it is suggested that you attend classes as much as possible, read the required lectures, teacher's notes regularly as all of them are foundations for the course. Lecture's notes are for supporting and not for submitting the reading material including the handouts. Try as much as possible to participate in classroom discussions, preparing the alignments given in the course.

14. Assessment scheme

Four unit exams will be given during the course of the year. Though each unit exam only covers material from the previous unit exam, there is some building of concepts that will show up on later exams. Problems on the exam will be similar to problems that exist in the textbook and on homework. As may be appropriate, some or all exams may be in a take home format. A cumulative final will also be given during finals week. So that the final grade will be based upon the following criteria:

Mid-term 1 Exam	15%
Mid-term 2 Exam	15%
Participation and Attendance	5%
Quiz	5%
Final Exam	60%
Total	100%

15. Student learning outcome:

The student will be able to analyse physical situations related to kinematics, Newton’s laws, oscillators, conservation laws and gravity and apply the techniques of vector analysis and differential equations to solve for motion in three dimensions, determine the effect that a no inertial system has on measurement, determine relationships between different physical quantities for orbital motion, and generate then solve the equations of motion from Lagrange's and Hamilton's equations. The student will appreciate the role that physics plays in everyday life and its relationship with other areas of study (chemistry, biology, Alquraan, Bible, history, etc.). The student will appreciate the importance of thinking through problems and using appropriate problem solving methods in every area of life.

16. Course Reading List and References:

Course Text: Required book

1- " Analytical Mechanics" , *Grant R. Fwoles.*
2- "Theoretical Mechanics", Schaum's outline series by *R. Spiegel*.
3- "Lagrangian Dynamics", Schaum's outline series by *Dare A. Wells*.

Additional Text: The following texts were requested to be put on reserve at the Physics Library:

a- Classical Mechanics, *Goldstein*
b- Mathematical Methods for Physicists, *Arfken*
c- Any other "Analytical Mechanics" textbook published in 21st century.
d-

17. The Topics:

Week	*Meeting*	*Ch.*	*Topics*
Mechanical Analysis I (First Semester)			
1	Oct. 2023	1	Vector Calculus Review
2	Oct. 2023	1	Change of Coordinates
3	Oct. 2023	2	Velo. & Accel. in Rectangular, Polar, Cylindri. & Spheri.
4	Oct. 2023	2	Newtonian Mechanics
5	Nov. 2023	2	Constant & Time Dependent Forces
6	Nov. 2023	2	Velocity & Position Dependent Forces
7	Nov. 2023	3	Linear Restoring forces & Harmonic Motion
8	Nov. 2023	3	Energy Considerations
9	Dec. 2023	3	Damped Harmonic Motion
10	Dec. 2023	3	Variation of Gravity with Height Vertical Motion
11	Dec. 2023	4	Vertical Motion
12	***Dec. 2023***	**-**	***Christmas Break-No Class***
13	Jan. 2024	4	Mid Term Exam
14	Jan. 2024	5	Review

Mechanical Analysis II (Second Semester)			
1	Feb. 2024	6	Conservative Force Field
2	Feb. 2024	6	General Motion of a Particle
3	Feb. 2024	7	Application of General Motion
4	Feb. 2024	7	Motion on the curve
5	Mar. 2024	7	System of Particles
6	Mar. 2024	7	Direct Collisions
7	***Mar. 2024***	-	***Newroz Break-No Class***
8	***Mar. 2024***	-	***Newroz Break-No Class***
9	Apr. 2024	8	Oblique Collisions
10	Apr. 2024	8	Kepler's Laws of Planetary Motion
11	Apr. 2024	9	Orbit of a Particle
12	Apr. 2024	9	Lagrangian Mechanics
13	May. 2024	9	Applications of Lagrange's Equations
14	May. 2024	9	Hamiltonian Mechanics
15	May. 2024	9	Connection Classical/Quantum Mechanics

18. Practical Topics (If there is any)

19. Extra notes:

Use this course in physics well. Physics is not purely knowledge, facts or information, but a skill. You need to practice this skill correctly in order to become proficient with it. 2 hours of study outside of class for each hour in class is recommended to be successful in this class. If you are weak in your calculus and problem solving skills you may need to invest even more time. I hope your appreciation for physics will grow during this course though it will require sustained amount of effort. Physics is a foundation for all other sciences since it attempts to quantify how this universe works from the largest to the smallest scales. The basic laws and structure present in the universe were created and are maintained by God and provide illustrations of God's majesty and power.

20. Examination:

Salahaddin University-Erbil

College of Science

Physics Department

3rd Stage/Second Semester

First Trial

Final Examination in:

*** Mechanical Analysis***

Date: June 2024

Period: 3 Hours

Q.1: Choose the ***correct*** answer of the following: *[10 Marks]*

i. For an elastic head-on collision between two bodies with *(m_1=3 kg, v_{1i}=5 m/s , m_2=1 kg* and *v_{2i}=3 m/s)*, the final speed of the second particle is:
(3 m/s , 4 m/s , 5 m/s , 6 m/s , None of them)

ii. For a particle moving in a circular path of radius *2m* with a velocity function *$v=2t^2 m/s$*, the magnitude of its total acceleration at *t = 1 s* is about:
(4 m/s^2 , 4.5 m/s^2 , 6 m/s^2 , 8.5 m/s^2 , None of them)

iii. Given that the velocity of a particle of mass *2 kg* in rectilinear motion is $\dot{x} = 2x^{-1}$. The force acting on the particle is:
(-8 x^{-3} , -4 x^{-3} , -8 x^{-2} , -4 x^{-2} , None of them)

iv. A mass on a spring oscillates back and forth with simple harmonic motion of amplitude *A*. At what point during its oscillation the kinetic energy is twice the potential energy?
($x = A/2$, $x = A/3$, $x = A/\sqrt{2}$, $x = A/\sqrt{3}$, *None of them*)

Q.2: *[5+5 Marks]*

(a) Express the vector $\vec{A} = 4\hat{e}_1' - 2\hat{e}_2'$ in terms of the $\hat{e}_1$ & $\hat{e}_2$ for rotation of the coordinate system through an angle 60°.

(b) Write the properties of the transformation matrix obtained in ***(a)***.

Q.3: What are the physical meanings of the following equations?

(I) $\vec{F}_i + \sum_{j=1}^{N} \vec{F}_{ij} = m_i \vec{a}_i$ (II) $2\ddot{x} + 4\dot{x} + 8x = 0$ (III) $\int_a^b \vec{F}.d\vec{r} = \int_a^b \vec{F}.d\vec{r}$ (Path1, Path2)

(IV) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{k}$ (V) $m\vec{a} = \vec{F} + \vec{R}$ [10 Marks]

Q.4: [5+5 Marks]

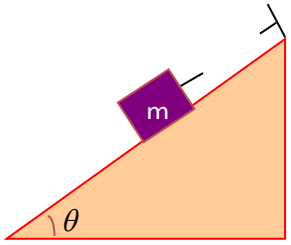
(a) Show that kinetic energy for a system of N-particles is: $T = \frac{1}{2} m v_{cm}^2 + \sum_{i=1}^{N} \frac{1}{2} m_i v_i'^2$

(b) Find the force for the potential energy function: $V = ce^{-(\alpha x + \beta y + \gamma z)}$

where $c, \alpha, \beta \text{ and } \gamma$ are constants.

Q.5: The orbit of a particle is a circle with the center of force on the circumference. The polar equation of the orbit is: $r = 2a\,Cos\theta$ where a is the radius of the circle. Find the law of the force by using energy equation of the orbit. [10 Marks]

Q.6: Find the acceleration of the following system by using Hamilton's equations for motion, where k is the stiffness of the spring. [10 Marks]



Good Luck *Dr. Tahseen G. Abdullah*

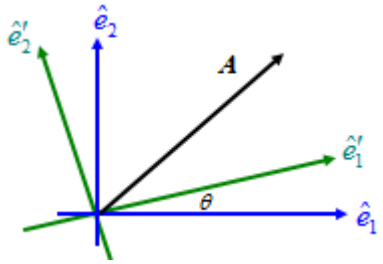
Ans. of Q.1:

i. *6 m/s* ii. *4.5 m/s²* iii. *-8 x⁻³* iv. $x = A/\sqrt{3}$

Ans. of Q.2:

(a)

$$\vec{A} = 4\hat{e}_1' - 2\hat{e}_2'$$



$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} \hat{e}_1'.\hat{e}_1 & \hat{e}_2'.\hat{e}_1 \\ \hat{e}_1'.\hat{e}_2 & \hat{e}_2'.\hat{e}_2 \end{bmatrix} \begin{bmatrix} A_1' \\ A_2' \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} \cos 60 & \cos(\frac{\pi}{2} + 60) \\ \cos(\frac{\pi}{2} - 60) & \cos 60 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 2+\sqrt{3} \\ 2\sqrt{3}-1 \end{bmatrix} = \begin{bmatrix} 3.732 \\ 2.464 \end{bmatrix}$$

$$\vec{A} = A_1\hat{e}_1 + A_2\hat{e}_2 = (2+\sqrt{3})\hat{e}_1 + (2\sqrt{3}-1)\hat{e}_2$$

(b)

Properties of transformation matrix

1- Magnitude of the vectors: *Invariant* under a rotation:

$$\vec{A} = A_1'\hat{e}_1' + A_2'\hat{e}_2' = 4\hat{e}_1' - 2\hat{e}_2'$$

$$\vec{A} = A_1\hat{e}_1 + A_2\hat{e}_2 = (2+\sqrt{3})\hat{e}_1 + (2\sqrt{3}-1)\hat{e}_2$$

$$\left|T\vec{A}\right| = \left|\vec{A}\right| = A = \sqrt{A_1^2 + A_2^2} = \sqrt{A_1'^2 + A_2'^2}$$

$$\left|\vec{A}\right| = \sqrt{4^2 + (-2)^2} = \sqrt{20}$$

$$\left|T\vec{A}\right| = \sqrt{(2+\sqrt{3})^2 + (2\sqrt{3}-1)^2} = \sqrt{4+3+4\sqrt{3}-4\sqrt{3}+12+1)} = \sqrt{20}$$

2- *T* for a reverse rotation (-*θ*) = $\tilde{T}$ (Transpose of *T*):

$$T(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \tilde{T}$$

3- $\tilde{T}T = I$, where *I* is the identity operator:

$$\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{4}+\frac{3}{4} & \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4}-\frac{\sqrt{3}}{4} & \frac{3}{4}+\frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Ans. of Q.3:

(I) Newton's second law for motion of particle *i* in a system of *N*-particles.

(II) Differential equation of under damped harmonic oscillator (***Since*** $\beta < w_o$ ***the case is under damping).***

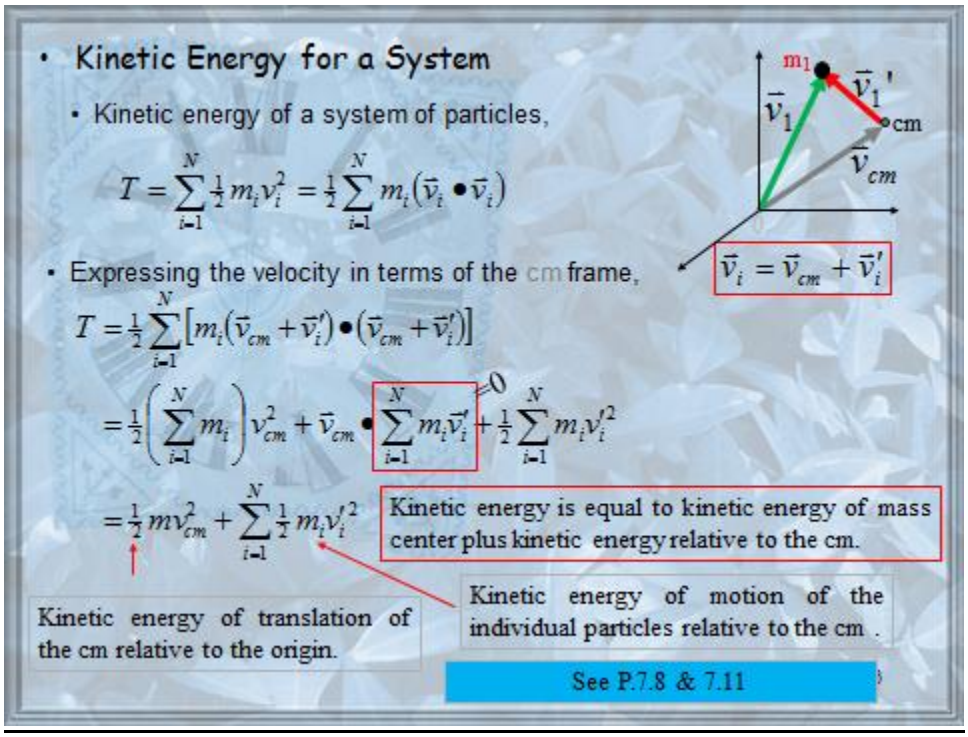
(III) Mathematical statement for a conservative force.

(IV) Velocity of a particle moving in the cylindrical coordinates.

(V) Differential Equation for Constrained Motion.

Ans. of Q.4:

(a)



(b)

$$\vec{F} = -\vec{\nabla} V = -(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z})$$

$$\vec{F} = -\vec{\nabla} V = ce^{-(\alpha x + \beta y + \gamma z)} \left(\hat{i}\alpha + \hat{j}\beta + \hat{k}\gamma \right)$$

Ans. of Q.5:

By using Energy eq. of the orbit:

$$\frac{1}{2}mh^2[(\frac{du}{d\theta})^2+u^2]+V(u^{-1})=E$$

$$r(\theta)=2aCos\theta \quad \Rightarrow \quad u=\frac{1}{r}=\frac{1}{2aCos\theta}$$

$$\frac{du}{d\theta}=\frac{1}{2a}\frac{Sin\theta}{Cos^2\theta}=2aSin\theta u^2$$

$$\frac{1}{2}mh^2[(2aSin\theta u^2)^2+u^2]+V(u^{-1})=E$$

$$\frac{1}{2}mh^2[(4a^2Sin^2\theta u^4)+u^2]+V(u^{-1})=E$$

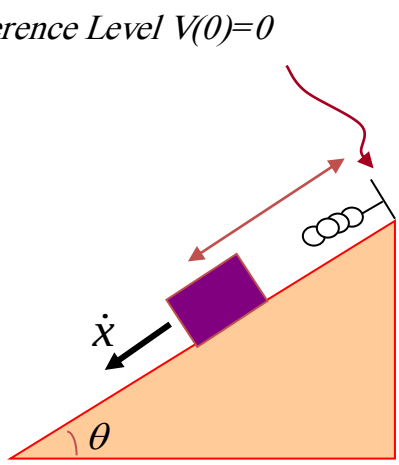
$$\frac{1}{2}mh^2[(4a^2-4a^2Cos^2\theta)u^4+u^2]+V(u^{-1})=E$$

$$\frac{1}{2}mh^2[4a^2u^4-u^2+u^2]+V(u^{-1})=E$$

$$V(r)=E-\frac{1}{2}mh^2[\frac{4a^2}{r^4}]\Rightarrow f(r)=-\frac{dV(r)}{dr}=-[0-\frac{1}{2}mh^2(-4a^2(4r^{-5}))]$$

$$Thus,\quad f(r)=\frac{-8a^2mh^2}{r^5}\Rightarrow\ f(r)\propto\frac{1}{r^5}$$

Ans. of Q.6:



- *The generalized coordinate is:* $q_1=x$
- *The kinetic energy for a system is:* $T = \frac{1}{2} m\dot{x}^2$

$$P = \frac{\partial T}{\partial \dot{x}} = m\dot{x} \rightarrow \dot{x} = \frac{p}{m} \rightarrow T = \frac{p^2}{2m}$$

- *The potential energy for a system is:*

$$V = -mgx\sin\theta + \frac{1}{2}k(x - x_0)^2$$

- ***The Hamiltonian function is:***

$$H = T + V = \frac{p^2}{2m} - mgx\sin\theta + \tfrac{1}{2}k(x - x_0)^2$$

$$\frac{\partial H}{\partial p} = \dot{x} \Rightarrow \frac{p}{m} = \dot{x} \Rightarrow p = m\dot{x} \Rightarrow \dot{p} = m\ddot{x}$$

$$\frac{\partial H}{\partial x} = -\dot{p} \rightarrow -mg\sin\theta + k(x - x_0) = -m\ddot{x}$$

$$\ddot{x} = g\sin\theta - \frac{k}{m}(x - x_0)$$