Chapter One

Elasticity

1.1 Stress and Strain:

- The **stress** in a material **is the ratio of the external force** acting through the material **divided by the cross-section area** through which the force is carried. **i.e.**;

$$stress\ (\sigma) = \frac{Force\ (F)}{area\ (A)} \ \dots (1) \quad \left[\frac{N}{m^2}\right] \quad Where\ F = mg$$

- The metric **unit of stress** is the Pascal (Pa).
- **One Pascal** is equal to **one Newton of force per square meter of area (1 N/m^2)**.

- The **strain is the amount a material has been deformed, divided by its original size**. **i.e.**;

$$Strain\ (\varepsilon) = \frac{\Delta L}{L_0} = \frac{x}{h} = -\frac{\Delta V}{V_0} \ \dots (2)$$

The result of a stress is strain, which is a measure of the degree of deformation.

It is found that, for sufficiently small stresses, **strain is proportional to stress ($\sigma \alpha \varepsilon$)**; the **constant of proportionality depends on the material being deformed** and on the nature of the deformation.

We call this **proportionality constant the elastic modulus**.

1.2 Elastic Modulus:

It is therefore defined as **the ratio of the stress to the resulting strain. i.e.;**

$$\boldsymbol{Elastic\ modulus}\ (\textbf{coefficient of elasticity}) = \frac{\boldsymbol{stress\ (\sigma)}}{\boldsymbol{Strain\ (\varepsilon)}} \quad \dots (3) \quad \left[\frac{\boldsymbol{N}}{\boldsymbol{m^2}}\right]$$

The proportionality of stress and strain (under certain conditions) is called Hooke's law, after **Robert Hooke** (**1635-1703**), a contemporary of Newton.

There are three relationships having the form of Equation (3), corresponding to tensile, shear, and bulk deformation, and all of them satisfy an equation similar to **Hooke's law** for springs:

$$\boldsymbol{F = -k\,\Delta x} \quad \dots (4)$$

where **F is the applied force**, **k is the spring constant**, and **Δx is the amount by which the spring is compressed**.

1.3 Elastic Limits:

The <u>**elastic limit**</u> of a substance is defined as **the maximum stress** that can be applied to the substance **before it becomes permanently deformed** and **does not return to its initial length**.

1.4 Three types of Elasticity:

We consider three types of deformation and define an elastic modulus for each:

- **(i)** **Young's modulus** or **Linear (tensile) elasticity** or **Elasticity of length**, which measures the resistance of a solid to a change in its length.
- **(ii)** **Shear modulus** or **Elasticity of shape** or **Modulus of Rigidity**, which measures the resistance to motion of the planes within a solid parallel to each other.
- **(iii)** **Bulk modulus** or **Elasticity of volume**, which measures the resistance of solids or liquids to changes in their volume.

1.4.1 Young's Modulus (Y):

Consider a long bar of **cross-sectional area A** and **initial length L_i** that is clamped at one end, as in Figure [1]. When an **external force is applied perpendicular to the cross section**, **internal forces in the bar resist distortion ("stretching")**, but the bar reaches an equilibrium situation in which its **final length L_f is greater than L_i** ($L_f > L_i$) and in which the external force is exactly balanced by internal forces.

In such a situation, the bar is said to be stressed. We define the **tensile stress** as **the ratio of the magnitude of the external force (F) to the cross-sectional area (A)**.

The **tensile strain** in this case is defined as **the ratio of the change in length (ΔL) to the original length L_i (L_0)**. We define **Young's modulus (Y)** by a combination of these two ratios:

$$Y = \frac{\text{tensile stress } (\sigma)}{\text{tensile strain } (\varepsilon)} = \frac{\frac{F_\perp}{A}}{\frac{\Delta L}{L_i}} \quad \dots (5) \left[\frac{N}{m^2}\right] \qquad \text{Where } A = \pi r^2 \text{ and } D = 2r$$

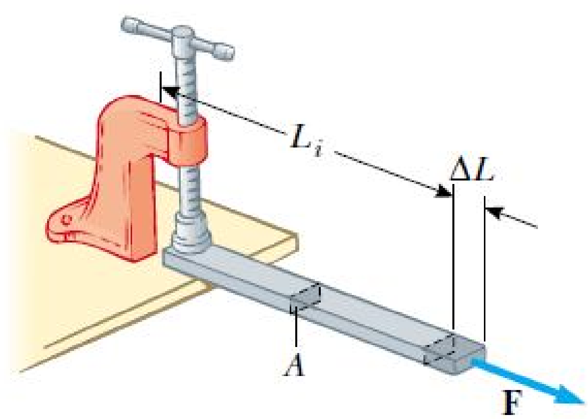

Figure [1]: A long bar clamped at one end is stretched by an amount ΔL under the action of a force F.

Young's modulus is typically **used to characterize a rod or wire** stressed under either tension or compression.

Typical values are given in table [1].

Table 1:

Information

Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

1.4.2 Shear Modulus (S):

Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force Figure [2-a]. The stress in this case is called a shear stress. If the object is originally a rectangular block, a shear stress results in a shape whose cross section is a parallelogram. A book pushed sideways, as shown in Figure [2-b], is an example of an object subjected to a shear stress. To a first approximation (for **small distortions**), **no change in volume** occurs with this deformation.

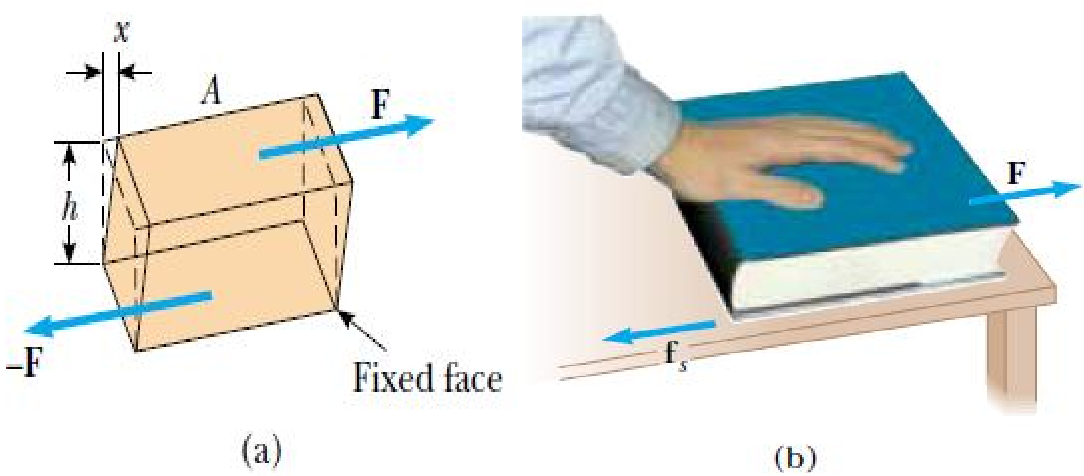

Figure [2]: (a) A sheared formation in which a rectangular block is distorted by two forces of equal magnitude but opposite directions applied to two parallel faces. (b) A book under shear stress.

We define the **shear stress** as F/A, **the ratio of the *tangential* force to the area A of the face being sheared**. The **shear strain** is defined as **the ratio x/h**, where **x is the horizontal distance** that the sheared face moves and **h is the height of the object**. In terms of these quantities, the **shear modulus** is:

$$S = \frac{shear\ stress\ (\sigma)}{shear\ strain\ (\varepsilon)} = \frac{\frac{F_{\parallel}}{A}}{\frac{x}{h}} \quad \dots (6) \quad \left[\frac{N}{m^2}\right]$$

Values of the shear modulus for some representative materials are given in table [1].

1.4.3 Bulk Modulus (B):

Bulk modulus characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object, as shown in Figure [3].

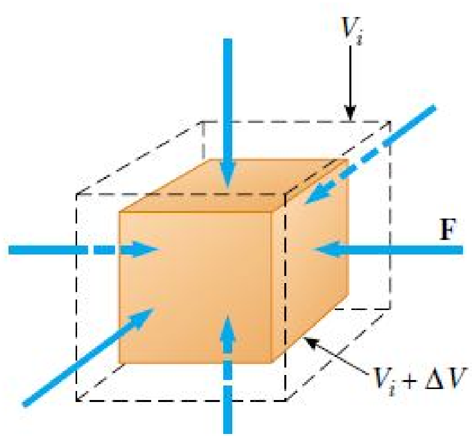

Figure [3]: When a solid is under uniform pressure, it undergoes a change in volume but no change in shape. This cube is compressed on all sides by forces normal to its six faces.

The **volume stress** is defined as **the ratio of the magnitude of the total force (F) exerted on a surface to the area (A) of the surface.** The quantity $P = F/A$ is called **pressure**. **If the pressure on an object change by an amount $\Delta P = \Delta F/A$, then the object will experience a volume change ΔV.**

The **volume strain** is equal to **the change in volume (ΔV) divided by the initial volume V_i (V_0)**.

Thus, from Equation (1), we can characterize a volume ("bulk") compression in terms of the **bulk modulus**, which is defined as:

$$B = -\frac{volume\ stress\ (\sigma)}{volume\ strain\ (\varepsilon)} = -\frac{\frac{\Delta F}{A}}{\frac{\Delta V}{V_i}} = -\frac{\Delta P}{\frac{\Delta V}{V_i}} \quad \dots (7) \quad \left[\frac{N}{m^2}\right]$$

A **negative sign** is inserted in this defining equation so that **B is a positive number**. This maneuver is necessary **because an increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa**.

Table [1] lists bulk moduli for some materials. If you look up such values in a different source, you often find that the reciprocal of the bulk modulus is listed.

The reciprocal of the bulk modulus is called the **compressibility** **of the material ($K = 1/B$)**.

Note from Table [1] that **both solids and liquids have a bulk modulus**. However, **no shear modulus and no Young's modulus are given for liquids because a liquid does not sustain a shearing stress or a tensile stress.** If a shearing force or a tensile force is applied to a liquid, the liquid simply flows in response.

Table 2 Compressibilities of Liquids

Liquid	Compressibility, k (Pa^{-1})	Compressibility, k (atm^{-1})
Carbon disulfide	93×10^{-11}	94×10^{-6}
Ethyl alcohol	110×10^{-11}	111×10^{-6}
Glycerine	21×10^{-11}	21×10^{-6}
Mercury	3.7×10^{-11}	3.8×10^{-6}
Water	45.8×10^{-11}	46.4×10^{-6}

1.5: Work done in Deforming a Body (W):

Whenever a body is deformed by the application of external forces, the body gets strained. The work done is stored in the body in the form of energy and is called the energy of strain.

Consider: **L: length of wire**, **l (ΔL): Change in the length**,

A: **area of cross section**, **F**: **applied force**, and **Y**: **Yong's modulus**.

Therefore, work done is:

$$\boldsymbol{W = F\,l}\quad [J] \rightarrow \quad dW = F\,dl \quad ...(8)$$

$$\int_0^W dW = \int_0^l F\,dl \qquad \text{But: } Y = \frac{F/A}{l/L_i} \quad \rightarrow \quad F = \frac{Y\,A\,l}{L_i}$$

$$W = \int_0^l \frac{Y\,A\,l}{L_i}\,dl \rightarrow W = \frac{Y\,A}{L_i}\int_0^l l\,dl$$

$$W = \frac{Y\,A}{L_i}\frac{l^2}{2} = \frac{1}{2}\frac{Y\,A\,l}{L_i}l = \frac{1}{2}Fl \quad ...(9)$$

Work done per unit volume (w):

$$w = \frac{W}{V} = \frac{\frac{1}{2}Fl}{A\,L_i} = \frac{1}{2}\frac{F}{A}\frac{l}{L_i} \rightarrow w\,\frac{1}{2}\sigma\varepsilon \quad ...(10) \quad \left[\frac{\boldsymbol{J}}{\boldsymbol{m^3}}\right]$$

Prove it!

1.6 Poisson Ratio (Relation between Elastic Constants) (ν):

Consider a unit cubic, which is subjected to outward **elongation force P** on each face. Let ν be the **Poisson's ratio for the material**.

In the table, the values of applied stress and the corresponding strain produced along the three perpendicular axes are shown. For a stress P the longitudinal strain produced $\frac{P}{Y}$ in its own direction and the corresponding strain in the other two perpendicular direction are $\frac{\nu P}{Y}$ and $-\frac{\nu P}{Y}$ **negative sign indicates contraction**.

Stress along			Strain along		
σ_x	σ_y	σ_z	ε_x	ε_y	ε_z
$+P$	0	0	$\frac{P}{Y}$	$-\frac{\nu P}{Y}$	$-\frac{\nu P}{Y}$
0	$+P$	0	$-\frac{\nu P}{Y}$	$\frac{P}{Y}$	$-\frac{\nu P}{Y}$
0	0	$+P$	$-\frac{\nu P}{Y}$	$-\frac{\nu P}{Y}$	$\frac{P}{Y}$
$+P$	$+P$	$+P$	$\sum \varepsilon_x = \frac{P}{Y}(1-2\nu)$	$\sum \varepsilon_y = \frac{P}{Y}(1-2\nu)$	$\sum \varepsilon_z = \frac{P}{Y}(1-2\nu)$

$$\therefore \varepsilon_V = \sum \varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{P}{Y}(1-2\nu) + \frac{P}{Y}(1-2\nu) + \frac{P}{Y}(1-2\nu) = \frac{3P}{Y}(1-2\nu)$$

But: $B = -\frac{volume\ stress\ (\sigma_V)}{volume\ strain\ (\varepsilon_V)} \rightarrow \varepsilon_V = \frac{P}{B}$

$$\varepsilon_V = \varepsilon_V \rightarrow \frac{3\cancel{P}}{Y}(1-2\nu) = \frac{\cancel{P}}{B}$$

$$\therefore B = \frac{Y}{3(1-2\nu)} \quad \dots(11)$$

Prove it!

Stress along			Strain along		
σ_x	σ_y	σ_z	ε_x	ε_y	ε_z
$+P$	0	0	$\frac{P}{Y}$	$-\frac{\nu P}{Y}$	$-\frac{\nu P}{Y}$
0	$-P$	0	$-\left(-\frac{\nu P}{Y}\right)$	$-\left(\frac{P}{Y}\right)$	$-\left(-\frac{\nu P}{Y}\right)$
$+P$	$-P$	0	$\sum \varepsilon_x = \frac{P}{Y}(1+\nu)$	$\sum \varepsilon_y = -\frac{P}{Y}(1+\nu)$	$\sum \varepsilon_z = 0$

$$\therefore \varepsilon_S = \sum \varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{P}{Y}(1+\nu) + \frac{P}{Y}(1+\nu) + 0 = \frac{2P}{Y}(1+\nu)$$

But: $S = \frac{shear\ stress\ (\sigma_S)}{shear\ strain\ (\varepsilon_S)} \rightarrow \varepsilon_S = \frac{P}{S}$

$$\varepsilon_S = \varepsilon_S \rightarrow \frac{2\cancel{P}}{Y}(1+\nu) = \frac{\cancel{P}}{S}$$

$$\therefore S = \frac{Y}{2(1+\nu)} \quad \ldots (12)$$

Prove it!

$B = \frac{Y}{3(1-2\nu)} \rightarrow 1 - 2\nu = \frac{Y}{3B}$ and $S = \frac{Y}{2(1+\nu)} \rightarrow 2 + 2\nu = \frac{Y}{S}$

Adding equations (1) & (2):

$$1 - 2\cancel{\nu} + 2 + 2\cancel{\nu} = \frac{Y}{3B} + \frac{Y}{S} \rightarrow 3 = \frac{Y}{3B} + \frac{Y}{S} \rightarrow \frac{3}{Y} = \frac{S+3B}{3BS}$$

$$9BS = YS + 3YB$$

From above equation, we can obtain:

$Y = \frac{9BS}{S+3B} \quad \ldots (13)$ Prove it!

$S = \frac{3YB}{9B-Y} \quad \ldots (14)$ Prove it!

$B = \frac{YS}{9S-3Y} \quad \ldots (15)$ Prove it!

From these equations the values of Y, S, and B can be calculated if any two values are known.

1.7 Theoretical Limiting Values of Poisson's Ratio:

We have:

$$\boldsymbol{B = \frac{Y}{3(1-2\nu)}} \quad and \quad \boldsymbol{S = \frac{Y}{2(1+\nu)}}$$

From these two equations: $Y = Y$

$$3B(1-2\nu) = 2S(1+\nu)$$

Since Poisson's ratio is positive quantity, and also B and S are always positive.

$$\therefore\ 1-2\nu > 0 \ \rightarrow\ 1 > 2\nu \rightarrow \frac{1}{2} > \nu$$

So, the values of $\boldsymbol{\nu}$ must be in the **ranges**:

$$\boldsymbol{0 < \nu < 0.5}$$

Another way:

Consider a wire of **length $\boldsymbol{L}$** and **diameter $\boldsymbol{D}$**. The wire is fixed at one end and a force is applied at the other end. Consequently, the length of the wire increases and the diameter of the wire decreases.

Suppose, **increase in length is $\boldsymbol{dL}$, decrease in diameter is $\boldsymbol{dD}$**.

$$\therefore\ \boldsymbol{\nu = -\frac{dD/D}{dL/L} = -\frac{dD}{dL}\frac{L}{D}} \quad \dots(16)$$

Initial volume of the wire is: $V = A\,L = \pi r^2 L = \frac{\pi D^2}{4} L$

Differentiating above equation: $dV = \frac{\pi}{4}(D^2\,dL + 2DL\,dD)$

If the volume of the wire remains unchanged after the force has been applied, then: $\boldsymbol{dV = 0}$.

$$\frac{\pi}{4}(D^2\,dL + 2DL\,dD) = 0 \ \rightarrow\ D^2\,dL = -2DL\,dD \ \rightarrow -\frac{dD}{dL}\frac{L}{D} = \frac{1}{2} \ \rightarrow\ \nu = \frac{1}{2}$$

Prove it!

This is the maximum possible value of Poisson's ratio.

Since the value of Poisson's ratio must be positive, so the **ranges of $\boldsymbol{\nu}$** will be: $\boldsymbol{0 < \nu < 0.5}$

1.8 Thermal expansion:

An **increase in the size** of a substance when the **temperature of the substance is increased**.

Thermal-expansion coefficient (α):

The proportionality constant that relates an **increase in length**, area, or volume to the original length, area, or volume as the result of an **increase in temperature**.

$$\alpha = \frac{1}{L}\frac{\Delta l}{\Delta t} \quad ...(17)$$

Note:

1 Newton (N) = 10^5 dyne

1 bound (lb) = 4.45 N

1Pascal (Pa) = 1 N/m^2 = 10 dyne/cm^2 = 1.45 x 10^{-4}lb/in^2.

1 atmosphere (atm) = 1.013 8 * 10^5 Pa.

1 atm = 1.01 x 10^5 Pa= 14.7 lb/in^2= 76 cm Hg

1in = 2.54 cm

1 Litter = 1 L = 1000 cm^3.

1 tons = 1000 Kg

Example 1: We analyzed a **cable** used to support an actor as he swung onto the stage. Suppose that the **tension** in the cable is $\mathbf{940\ N}$ as the actor reaches the lowest point. What **diameter** should a **10-m-long** steel wire have if we do not want it to stretch **more than** $\mathbf{0.5\ cm}$ under these conditions? $(If\ Known\ \mathbf{Y = 20 \times 10^{10} N/m^2})$.

Solution:

$$Y = \frac{F/A}{\Delta L/L_i}$$

$$A = \frac{FL_i}{Y\Delta L} = \frac{(940\ \text{N})(10\ \text{m})}{(20 \times 10^{10}\ \text{N/m}^2)(0.005\ \text{m})}$$

$$= 9.4 \times 10^{-6}\ \text{m}^2$$

Because $A = \pi r^2$, the radius of the wire can be found from

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{9.4 \times 10^{-6}\ \text{m}^2}{\pi}} = 1.7 \times 10^{-3}\ \text{m} = 1.7\ \text{mm}$$

$$d = 2r = 2(1.7\ \text{mm}) = \boxed{3.4\ \text{mm}}$$

Example 2: A **steel** rod **2m long** has a cross-sectional **area** of **0.3cm^2**. The rod is now hung by one end from a support structure, and a **550kg** milling machine is hung from the rod's lower end. Determine the **stress**, the **strain**, and the **elongation** of the rod. $(If\ Known\ \mathbf{Y = 20 \times 10^{10} Pa})$.

Solution:

$$\text{Stress} = \frac{F}{A} = \frac{(550\ \text{kg})(9.8\ \text{m/s}^2)}{3.0 \times 10^{-5}\ \text{m}^2} = 1.8 \times 10^{8}\ \text{Pa}$$

$$\text{Strain} = \frac{\Delta L}{L_i} = \frac{\text{Stress}}{Y} = \frac{1.8 \times 10^{8}\ \text{Pa}}{20 \times 10^{10}\ \text{Pa}} = 9.0 \times 10^{-4}$$

$$\text{Elongation} = \Delta L = (\text{Strain}) \times L_i = (9.0 \times 10^{-4})(2.0\ \text{m})$$

$$= 0.0018\ \text{m} = 1.8\ \text{mm}$$

Example 3: A **square steel** plate is **10.0 cm on a side** and **0.500 cm thick**. (a) Find the shear strain that results if a **force** of magnitude $\mathbf{9 \times 10^5\ N}$ is applied to each of the four sides, parallel to the side. (b) Find the **displacement** $\boldsymbol{x}$ in centimeters. ($If\ Known\ \boldsymbol{S} = \mathbf{7.5 \times 10^{10} Pa}$).

Solution:

$F_{\parallel} = 9.0 \times 10^5$ N.

$h = 0.100$ m.

$S = 7.5 \times 10^{10}$ Pa for steel.

$A = (0.100\text{ m})(0.500 \times 10^{-2}\text{ m})$.

$$\text{(a) Shear strain} = \frac{F_{\parallel}}{AS} = \frac{(9 \times 10^5\text{ N})}{[(0.100\text{ m})(0.500 \times 10^{-2}\text{m})][7.5 \times 10^{10}\text{ Pa}]} = 2.4 \times 10^{-2}.$$

$$\text{(b)}\quad x = (\text{Shear strain}) \cdot h = (0.024)(0.100\text{ m}) = 2.4 \times 10^{-3}\text{m}.$$

Example 4: A solid brass sphere is **initially** surrounded by air, and the air **pressure** exerted on it is $\mathbf{1.0 \times 10^5\ N/m^2}$ (normal atmospheric pressure). The sphere is lowered into the ocean to a **depth where the pressure is** $\mathbf{2.0 \times 10^7\ N/m^2}$. The **volume** of the sphere in air is $\mathbf{0.50\ m^3}$. By how much does this **volume change** once the sphere is submerged?

Solution:

$$B = -\frac{\Delta P}{\Delta V / V_i} \longrightarrow \Delta V = -\frac{V_i \Delta P}{B}$$

$$\Delta V = -\frac{(0.50\text{ m}^3)(2.0 \times 10^7\text{ N/m}^2 - 1.0 \times 10^5\text{ N/m}^2)}{6.1 \times 10^{10}\text{ N/m}^2}$$

$$= -1.6 \times 10^{-4}\text{ m}^3$$

The negative sign indicates that the volume of the sphere decreases.

Example 5: The **Young's modulus** of a metal is $\mathbf{2 \times 10^{11} N/m^2}$ and its breaking **stress** is $\mathbf{1.078 \times 10^{9} N/m^2}$. Calculate the maximum amount of **energy per unit volume** which can be stored in the metal when stretched.

Solution:

Here $\mathrm{Y} = 2 \times 10^{11} \mathrm{N/m^2}$

Maximum stress: $\sigma = \mathbf{1.078 \times 10^{9} N/m^2}$

Energy per unit volume:

$$w = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\sigma\frac{\sigma}{Y}$$

$$w = \frac{1}{2}\frac{(1.078 \times 10^9)^2}{2 \times 10^{11}} = 2.9 \times 10^6 J/m^3$$

Example 6: The **modulus of rigidity** *and* **Poisson's ratio** of the material of a wire are $\mathbf{2.87 \times 10^{10} N/m^2}$ and **0.379** respectively. Find the value of **young's modulus** of the material of the wire.

Solution:

$$S = \frac{Y}{2(1+\nu)}$$

$$Y = 2S(1+\nu)$$

$$Y = 2 \times 2.87 \times 10^{10}(1 + 0.379)$$

$$Y = 7.915 \times 10^{10} N/m^2.$$

Example 7: A **copper** wire **3 m long** for which **Young's modulus** is $\mathbf{12.5 \times 10^{11} D/cm^2}$, has a **diameter** of **1 mm**. If a **weight** of **10 Kg** is attached to one end, what extension is produced? If **Poisson's ratio 0.26**, what **lateral compression** is produced?

Solution:

Here $\boldsymbol{L = 3\,m = 300\,cm}$, $\boldsymbol{Y = 12.5 \times 10^{11} D/cm^2}$, $\boldsymbol{r = 0.5\,mm = 0.05\,cm}$

$$A = \pi r^2 = \pi \times (0.05)^2 cm^2$$

$$F = 10Kg \times 9.8 = 10000g \times 980 = 9.8 \times 10^6 D$$

$$Y = \frac{F \times L}{A \times l} \Rightarrow l = \frac{F \times L}{A \times Y}$$

$$l\ (\Delta L) = \frac{(9.8 \times 10^6) \times (300)}{(\pi \times (0.05)^2) \times (12.5 \times 10^{11})} = 0.2997cm$$

$$Poission'sratio = \frac{lateralstrain}{longitudinalstrain}$$

$$\nu = \frac{d/D}{l/L} \Rightarrow d = \frac{\nu Dl}{L}$$

$$d = \frac{(0.26)(0.1)(0.2997)}{(300)} = 2.598 \times 10^{-5}cm$$

Example 8: A steel wire **8 m long** and **4 mm in diameter** is fixed to two rigid supports. Calculate **increase in tension** when the **temperature** falls by $\mathbf{10\ ^oC}$. If Known ($\boldsymbol{\alpha = 12 \times 10^{-6} /\ ^oC}$), ($\boldsymbol{Y = 2 \times 10^{11} N/m^2}$).

Solution:

Increase in length $\Delta l = L\alpha\Delta t$

Strain $= \varepsilon = \frac{\Delta l}{L} = \frac{\cancel{L}\alpha\Delta t}{\cancel{L}} = \alpha\Delta t$

Stress $= \sigma = Y \times \varepsilon = Y\alpha\Delta t$

Area of cross section $= A = \pi r^2$

$F = \sigma \times A$

Increase in tension $= F = Y\alpha\Delta t\pi r^2$

$$= (2 \times 10^{11}) \times (12 \times 10^{-6}) \times (10) \times \left(\frac{22}{7}\right) \times (2 \times 10^{-3})^2 = 301.7N^2$$

Home Work:

Q1: A relaxed biceps muscle requires a force of 25.0 N for an elongation of 3.0 cm; the same muscle under maximun tension requires a force of 500 N for the same elongation.Find Young's modulus for the muscle tissue under each of theseconditions if the muscle is assumed to be a uniform cylinder withlength 0.200 m and cross-sectional area 50.0 cm^2.

Solution:

Q2: A hydraulic press contains $0.25\ m^3$ $(250\ L)$ of oil. Find the decrease in the volume of the oil when it is subjected to a pressure increase $\Delta p = 1.6 \times 10^7 Pa$ (about 160 atm or 2300 psi). The bulk modulus of the oil is $B = 5.0 \times 10^9\ Pa$ (about $5.0 \times 10^4\ atm$), and its compressibility is $k = 1/B = 20 \times 10^{-6}\ atm^{-1}$.

Solution:

Q3: Find the work done in stretching a wire of 1sq mm cross section and 2m long through 0.1mm. ($Y = 2 \times 10^{11} N/m^2$).

Solution:

Q 4: Calculate the Poisson's ratio for the material, given $Y = 12.25 \times 10^{10} N/m^2$, and $S = 4.55 \times \frac{10^{10} N}{m^2}$.

Solution:

Q 5: A steel rod of length 5 m is fixed rigidly between two supports. The coefficient of linear expansion of steel $= 12 \times 10^6 / {}^oC$. Calculate the stress in the rod for an increase in temperature of $40\ {}^oC$. The Young's modulus of elasticity of steel $= 2 \times 10^{11} N/m^2$.

Solution: