Chapter Two

Fluid Mechanics

2.1 Introduction:

A **fluid** is a **collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by the walls of a container**. Both **liquids** and **gases** are **fluids**.

2.2 Density:

An important property of any material is its **density**, defined as its **mass per unit volume**. A homogeneous material such as **ice** or **iron** has the **same density** throughout. If a **mass (m)** of homogeneous material has **volume (V)**, the **density (ρ)** is:

$$\rho = \frac{m}{V} \quad \dots (1)$$

Two objects made of the **same material** *have* the **same density** even though they may *have* **different masses** and **different volumes**. That's because the ratio of mass to volume is the same for both objects (Fig.1).

Different mass, same density: Because the **wrench** and **nail** are **both made of steel**, they have the **same density**.

Figure [1]: Two objects with different masses and different volumes but the same density.

Table 1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

The **SI unit of density is the kilogram per cubic meter** ($1\ kg/m^3$).

The **cgs unit of density is the gram per cubic centimeter** ($1\ g/cm^3$), is also widely used:

$$1\ g/cm^3 = 1000\ kg/m^3$$

1.3 Pressure in a Fluid:

When a **fluid (either liquid or gas)** is at rest, it exerts a **force perpendicular to any surface** in contact with it, such as a container wall or a body immersed (dipped) in the fluid.

Consider a **small surface of area dA** centered on a point in the fluid; the normal force exerted by the fluid on each side is $dF_{\perp}$ (Fig. 2). We define the **pressure (p)** at that point as **the normal force per unit area-that is, the ratio of $dF_{\perp}$ to dA** (Fig. 3):

$$p = \frac{dF_{\perp}}{dA} \quad \dots (2)$$

If the **pressure is the same at all points** of a finite plane surface with area, then:

$$p = \frac{F_{\perp}}{A} \quad \dots (3)$$

where **$F_{\perp}$ is the net normal force on one side of the surface**. The **SI unit of pressure is the Pascal**, where:

1Pascal (Pa) = 1 N/m^2.

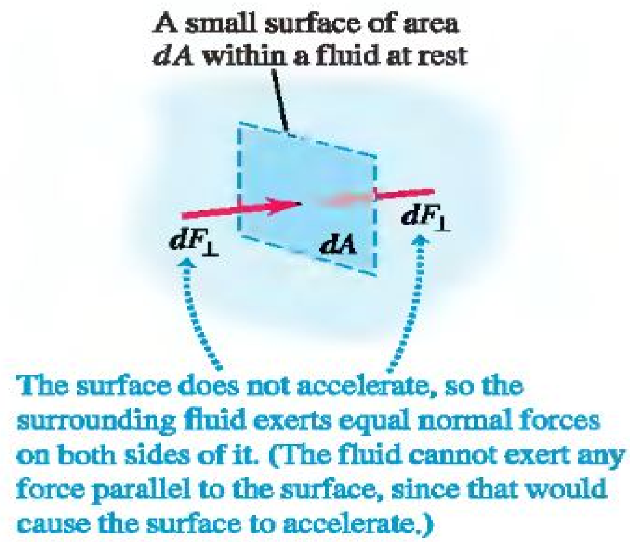


Figure [2]: Forces acting on a small surface within a fluid at rest.

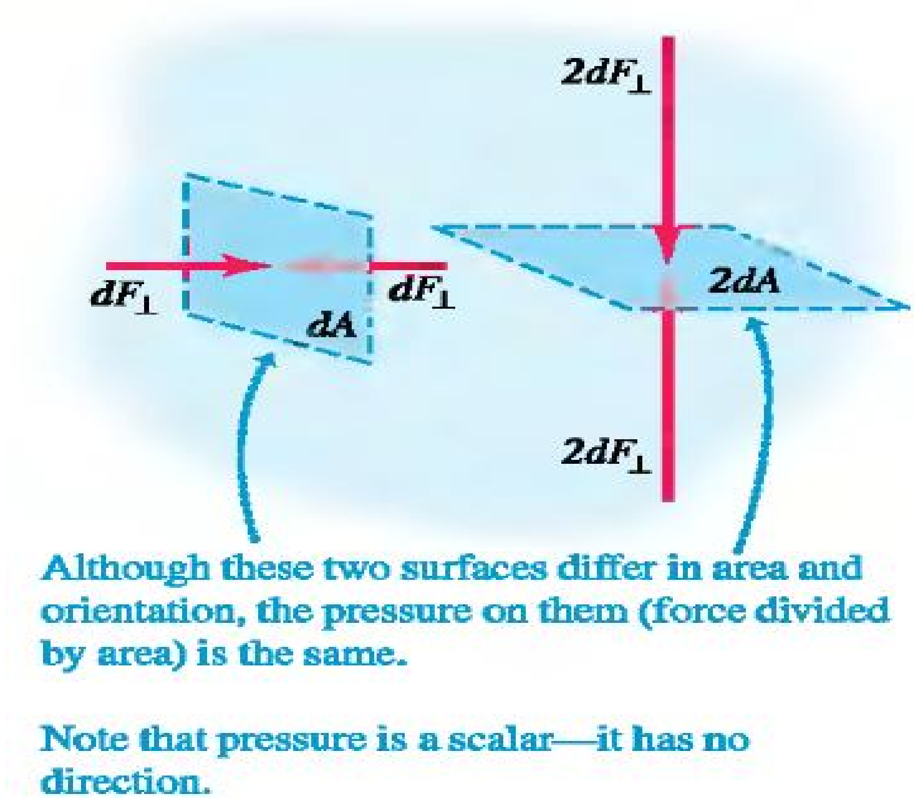


Figure [3]: The pressure on either side of a surface is force divided by area. Pressure is a scalar with units of Newton's per square meter. By contrast, force is a vector with units of Newton's.

Atmospheric pressure (p_{atm}) is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live.

This pressure varies with weather changes and with elevation.

Normal atmospheric pressure at sea level (an average value) **is 1 atmosphere (atm),** defined to be exactly **101,325 Pa**.

To four significant figures;

$$(p_a)_{av} = 1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$$

Save it!

$$= 1.013\ \text{bar} = 1013\ \text{millibar} = 14.70\ \text{lb/in.}^2$$

1.3.1 Pressure, Depth, and Pascal's Law:

If the **weight** (w) of the fluid can be **neglected**, the **pressure** in a fluid is the **same** throughout its **volume**.

Consider a thin element of fluid with **thickness** dy (Figure 4a). The bottom and top surfaces each have area (A), and they are at elevations y **and** $y + dy$ above some reference level where $y = 0$. The **volume of the fluid element is** $dV = A\,dy$, its **mass is** $dm = \rho\,dV = \rho A\,dy$, and its **weight is** $dw = dmg = \rho gAdy$.

What are the other forces on this fluid element (Fig 4b)**?**

Call the **pressure** ***at the bottom surface*** **is** p; **the** total y-component of **upward force on this surface is** $+pA$. The **pressure** ***at the top surface*** **is** $p + dp$, and the total y-component of **downward force on the top surface is** $-(p + dp)A$.

The fluid element is **in equilibrium**, so the **total y-component of force**, including the weight and the forces at the bottom and top surfaces, must be zero:

$$\sum F_y = F_{bottom} + F_{top} + w = 0 \rightarrow pA - (p + dp)A - \rho gAdy = 0$$

$$\cancel{pA} - \cancel{pA} - Adp - \rho gAdy = 0 \rightarrow \frac{\cancel{A}dp}{\cancel{A}dy} = -\frac{\rho g\cancel{A}\cancel{dy}}{\cancel{A}\cancel{dy}}$$

When we **divide** out the **area** (A) and **rearrange** (dy), we get: $\frac{dp}{dy} = -\rho g$ ... (4) Prove it!

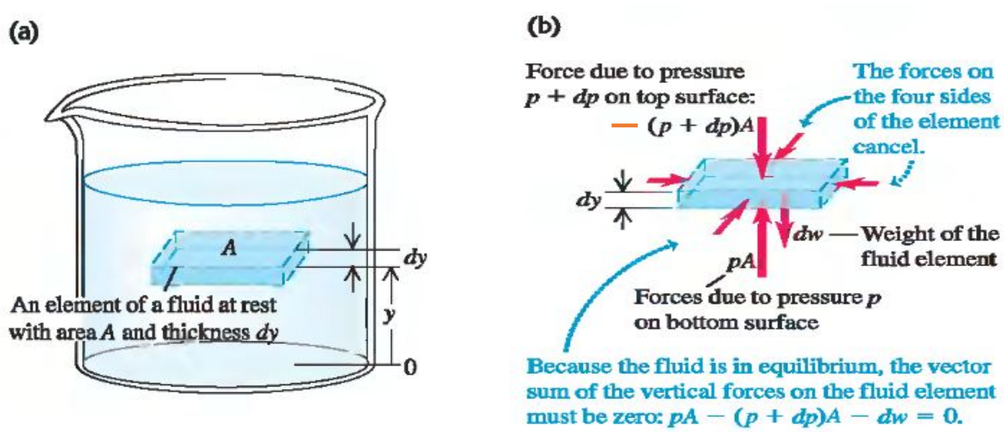


Figure [4]: The forces on an element of fluid in equilibrium.

Equation (4) shows that when $\boldsymbol{y}$ **increases**, $\boldsymbol{p}$ **decreases**; that is, as we move upward in the fluid, pressure decreases, as we expect.

If $\boldsymbol{p_1}$ **and** $\boldsymbol{p_2}$ **are the pressures** at **elevations** $\boldsymbol{y_1}$ **and** $\boldsymbol{y_2}$ respectively and if $\boldsymbol{\rho}$ **and** $\boldsymbol{g}$ **are constant**, then:

$$\frac{dp}{dy} = -\rho g \rightarrow \frac{p_2 - p_1}{y_2 - y_1} = -\rho g \rightarrow \boldsymbol{p_2 - p_1 = -\rho g(y_2 - y_1)} \ \dots (\mathbf{5})$$

It's often convenient to express Eq. (5) in terms of the depth below the surface of a fluid (Fig. 5). Take point 1 at any level in the fluid and let $\boldsymbol{p}$ represent the pressure at this point. Take point 2 at the surface of the fluid, where the **pressure is $\boldsymbol{p_0}$ (subscript zero for zero depth)**. The **depth of point 1 below the surface is $\boldsymbol{h = y_2 - y_1}$** and Eq. (5) becomes:

$$p_1 = p_0 \ and \ \ h = y_2 - y_1 \rightarrow \boldsymbol{p_2 = p_0 + \rho g h} \ \dots (\mathbf{6})$$

Prove it!

This equation represented of **the pressure in a fluid of uniform density**.

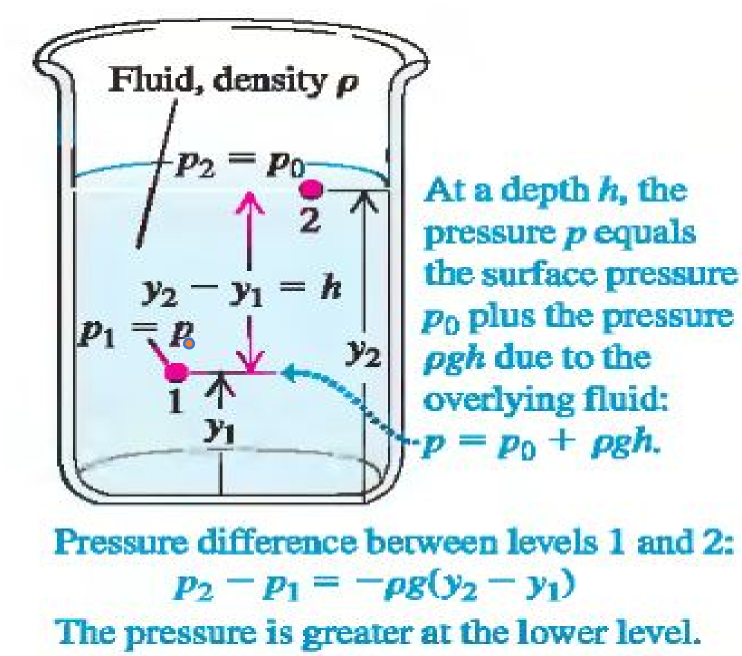


Figure [5]: How pressure varies with depth in a fluid with uniform density.

The **pressure (p) at a depth h** is <u>**greater than**</u> **the pressure (p_0)** at the surface by an amount $\boldsymbol{pgh}$.

Note that the **pressure** is the same at any two points **at the same level in the fluid**. The **shape** of the container **does not matter (important)** (Fig. 6).

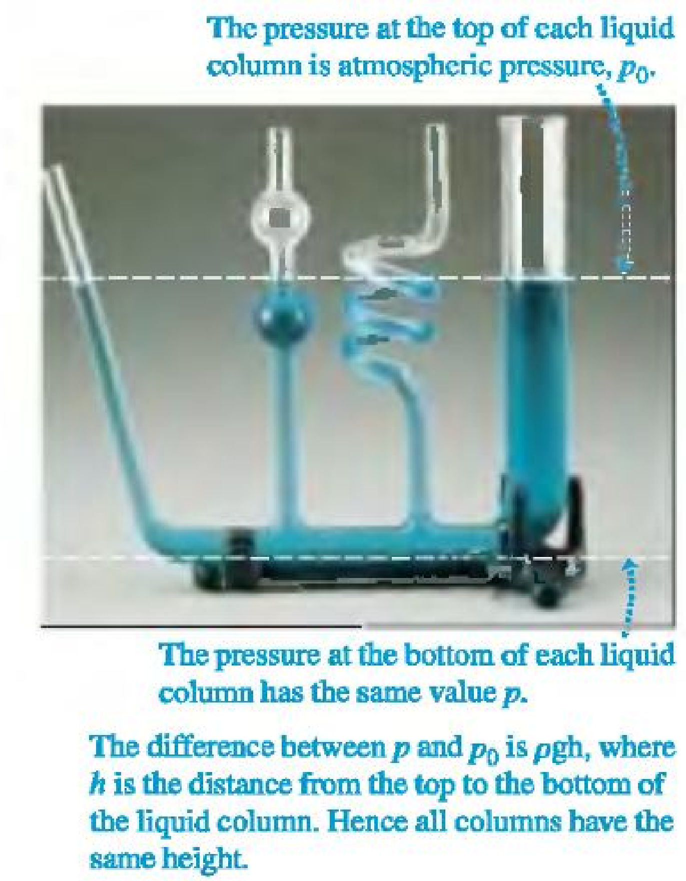


Figure [6]: Each fluid column has the same height, no matter what its shape.

Equation (6) shows that <u>if</u> we **increase the pressure p_0** at the top surface, possibly by using a piston that fits tightly inside the container to push down on the fluid surface, the **pressure p at any depth increases** by exactly the **same amount**.

This fact was recognized in **1653** by the **French** scientist **Blaise Pascal** (1623-1662) and is called **Pascal's law**.

<u>Pascal's law</u>: **a change in the pressure applied to a fluid is transmitted undiminished to every point** of the fluid and to the walls of the container.

Application:

The hydraulic lift shown schematically in Figure [7] illustrates Pascal's law. A **piston** with **small cross-sectional area** A_1 exerts a **force** F_1 on the surface of a liquid such as oil. The applied **pressure** $p = \frac{F_1}{A_1}$, is transmitted through the connecting pipe to a **larger piston** of **area** A_2. The applied **pressure is the same in both cylinders**, so:

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad and \quad F_2 = \frac{A_2}{A_1} F_1 \quad \dots (7)$$

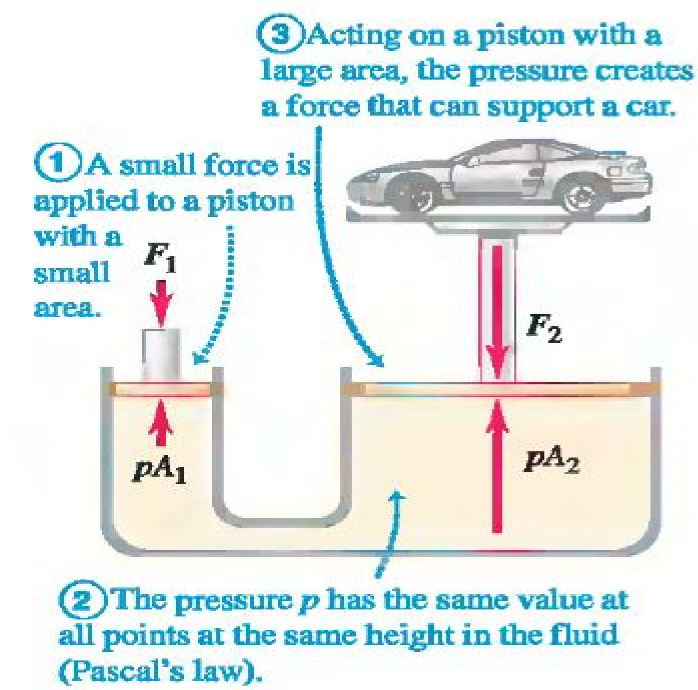


Figure [7]: The hydraulic lift is an application of Pascal's law. The size of the fluid-filled container is exaggerated for clarity.

The **hydraulic lift** is a force-multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. **Dentist's chairs**, **car lifts and jacks**, many **elevators**, and **hydraulic brakes all use this principle**.

2.3.2 Absolute Pressure and Gauge Pressure:

The **excess pressure above atmospheric pressure is usually called gauge pressure, and** the **total pressure is called absolute pressure.**

The **simplest pressure gauge** is the **open-tube manometer** (Fig. 8-a). The **U-shaped tube contains a liquid** of density ρ, **often mercury or water**. **The left end of the tube is connected to the container** where the pressure p is to be measured, **and the right end is open to the atmosphere at pressure** $p_0 = p_{atm}$.

The **pressure at the bottom of the tub**e due to the fluid in the **left** column is $p + \rho g y_1$ and the **pressure at the bottom** due to the fluid in the **right** column is $p_{atm} + \rho g y_2$. These **pressures** are measured at the **same level**, so they must be equal ($p_{left} = p_{right}$):

$$p + \rho g y_1 = p_{atm} + \rho g y_2$$

$$p - p_0 = \rho g(y_2 - y_1) \rightarrow p - p_0 = \rho g h \quad \dots (8)$$

In Eq. (8), **p is the absolute pressure**, and the **difference $p - p_{atm}$ between absolute and atmospheric pressure** is the **gauge pressure**. The **gauge pressure** is **proportional** to the **difference in height $h = y_2 - y_1$** of the liquid columns.

Another **common pressure gauge** is the **mercury barometer**. It **consists of a long glass tube, closed at one end that has been filled with mercury and then inverted in a dish of mercury** (Fig. 8-b).

The space above the mercury column contains only **mercury vapor**; its **pressure is negligibly small**, so the **pressure p_0 at the top of the mercury column is practically zero**. From Eq. (6): $p = p_0 + \rho g h \quad \dots (9)$ Prove it!

Thus, the mercury barometer reads the atmospheric pressure p_{atm} directly from the height of the mercury column.

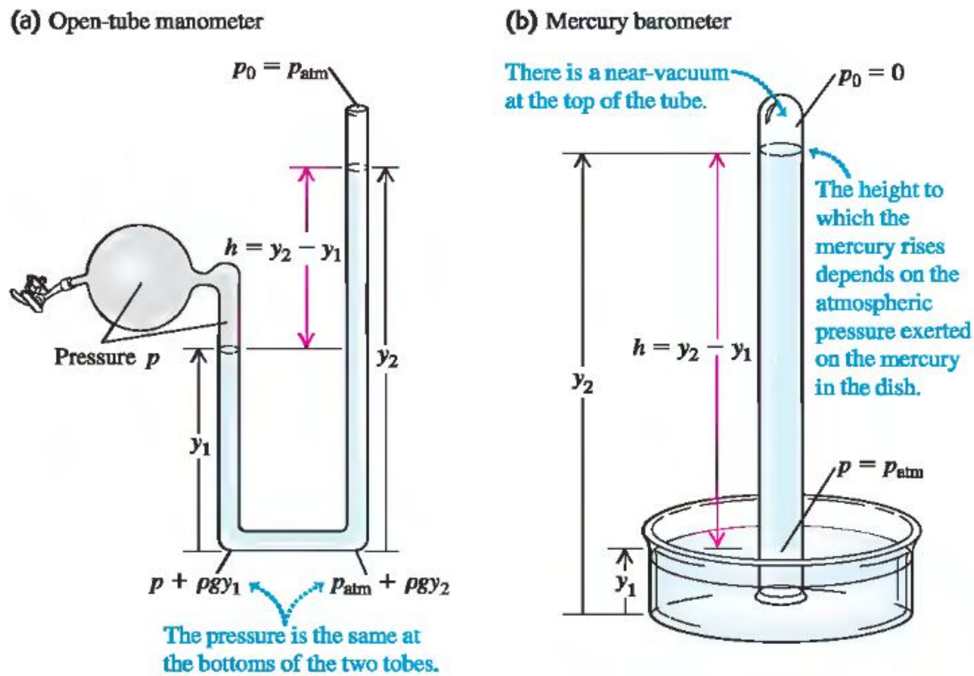


Figure [8]: Two types of pressure gauge.

2.4 Buoyant Forces and Archimedes' Principle:

Buoyancy is a familiar phenomenon: **A body immersed in water seems to weigh less than when it is in air**. When the body is less dense than the fluid, it floats. The **human body usually floats in water**, and a **helium-filled balloon floats in air**.

Archimedes
Greek Mathematician, Physicist, and Engineer (287–212 B.C.)

Archimedes' principle states: **When a body is completely or partially immersed in a fluid,** the fluid exerts **an upward force on the body equal to the weight ($B = F_g$)** of the fluid displaced by the body.

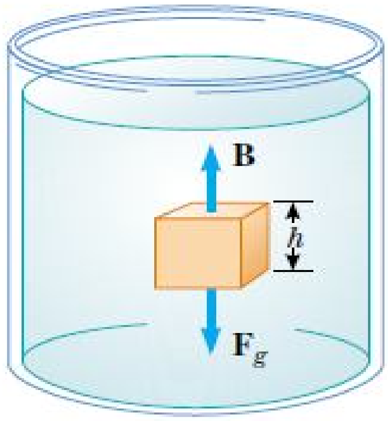


Figure [9]: The external forces acting on the cube of liquid are the gravitational force F_g and the buoyant force B. Under equilibrium conditions, $B = F_g$.

To understand the origin of the buoyant force, consider a cube immersed in a liquid as in Figure [9]. The **pressure p_b at the bottom** of the cube is **greater than** the **pressure p_t at the top** by an amount $\rho_{fluid}gh$, where ***h* is the height of the cube** and **ρ_{fluid} is the density of the fluid**. The pressure at the bottom of the cube causes an **upward force equal to p_bA**, where ***A* is the area of the bottom face**. The pressure at the top of the cube causes a **downward force equal to p_tA**.

The resultant of these two forces is the **buoyant force *B***:

$$B = F_b - F_t = p_bA - p_tA = (p_b - p_t)A = \rho_{fluid}ghA \rightarrow B = \rho_{fluid}gV \quad \dots(10)$$

where ***V* is the volume of the fluid** displaced by the cube. Because the product **$\rho_{fluid}V$ is equal to the mass** of fluid displaced by the object, we see that:

$$B = mg \quad \dots(11)$$

Prove it!

where ***mg* is the weight of the fluid** displaced by the cube.

Example 1: Find the **mass** and **weight** of the **air** in a living room at 20°C with a $\mathbf{4.0\ m \times 5.0\ m}$ floor and a ceiling $\mathbf{3.0\ m}$ **high**. If Kown $\boldsymbol{\rho_{air} = 1.2 Kg/m^3}$.

I) What are the **mass** and **weight** of an equal volume of **water**?

II) What is the **total downward force** on the surface of the floor due to **air pressure of 1.00 atm**?

Solution:

The volume of the room is $V = (3.0\,\mathrm{m})(4.0\,\mathrm{m}) \times (5.0\,\mathrm{m}) = 60\,\mathrm{m}^3$.

The mass m_{air} of air is

$$m_{air} = \rho_{air} V = (1.20\,\mathrm{kg/m^3})(60\,\mathrm{m^3}) = 72\,\mathrm{kg}$$

The weight of the air is

$$w_{air} = m_{air} g = (72\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 700\,\mathrm{N}$$

The mass of an equal volume of water is

$$m_{water} = \rho_{water} V = (1000\,\mathrm{kg/m^3})(60\,\mathrm{m^3}) = 6.0 \times 10^4\,\mathrm{kg}$$

The weight is

$$w_{water} = m_{water} g = (6.0 \times 10^4\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 5.9 \times 10^5\,\mathrm{N}$$

The floor area is $A = (4.0\,\mathrm{m})(5.0\,\mathrm{m}) = 20\,\mathrm{m}^2$,
the total downward force is

$$F_\perp = pA = (1.013 \times 10^5\,\mathrm{N/m^2})(20\,\mathrm{m^2})$$
$$= 2.0 \times 10^6\,\mathrm{N} = 4.6 \times 10^5\,\mathrm{lb} = 230\ \mathrm{tons}$$

Example 2: The mattress of a **water** bed is **2 m long by 2 m wide** and **30 cm deep**.

A: Find the **weight** of the water in the mattress.

B: Find the **pressure** exerted by the water on the floor when the bed rests in its normal position. Assume that the entire lower surface of the bed makes contact with the floor.

Solution:

$$A = L * W = 4\ m^2 \ and\ V = Ah = 1.2\ m^3$$

$$m = \rho V = (1\,000\ \mathrm{kg/m^3})(1.20\ \mathrm{m^3}) = 1.20 \times 10^3\ \mathrm{kg}$$

$$F = m\,g = (1.20 \times 10^3\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 1.18 \times 10^4\ \mathrm{N}$$

$$P = \frac{F}{A} = \frac{1.18 \times 10^4\ \mathrm{N}}{4.00\ \mathrm{m^2}} = 2.95 \times 10^3\ \mathrm{Pa}$$

Example 3: A storage tank **12 m deep** is filled with **water**. The top of the tank is **open to the air**. What is the **absolute pressure** at the bottom of the tank, and the **gauge pressure**?

Solution:

$h = 12.0\,\text{m}$

p_0 equals 1 atm $= 1.01 \times 10^{-5}$ Pa

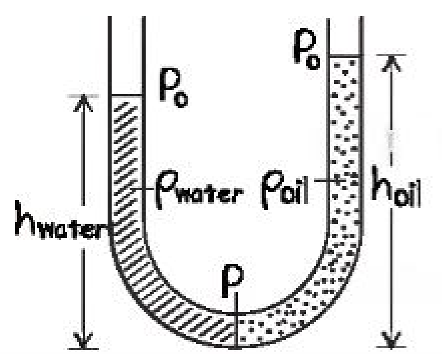
$$p = p_0 + \rho g h$$
$$= (1.01 \times 10^5\,\text{Pa}) + (1000\,\text{kg/m}^3)(9.80\,\text{m/s}^2)(12.0\,\text{m})$$
$$= 2.19 \times 10^5\,\text{Pa} = 2.16\,\text{atm}$$

The gauge pressure is

$$p - p_0 = (2.19 - 1.01) \times 10^5\,\text{Pa}$$
$$= 1.18 \times 10^5\,\text{Pa} = 1.16\,\text{atm}$$

Example 4: A manometer tube is partially filled with **water**. **Oil** (which does not mix with water and has a lower density than water) is then poured into the left arm of the tube until the oil-water interface is at the **midpoint of the tube**. Both arms of the tube are open to the air. Find a relationship between the **heights h_{oil} and h_{water}**.

Solution:



$$p - p_0 = \rho_{water} g h_{water}$$

$$p - p_0 = \rho_{oil} g h_{oil}$$

$$\because p - p_0 = p - p_0$$

$$\rho_{water} g h_{water} = \rho_{oil} g h_{oil}$$

$$\frac{h_{oil}}{h_{water}} = \frac{\rho_{water}}{\rho_{oil}}$$

Example 5: Estimate the **force** exerted on **your eardrum** due to the **water** above when you are swimming at the bottom of a pool that is **5.0 m deep**.

Solution:

$$P_{bot} - P_0 = \rho g h$$
$$= (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(5.0 \text{ m})$$
$$= 4.9 \times 10^4 \text{ Pa}$$

We estimate the surface area of the eardrum to be approximately $1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$.

This means that the force on it is $F = (P_{bot} - P_0)A \approx 5$ N.

Example 6: In a car lift used in a service station, compressed **air** exerts a force on a **small piston** that has a circular cross section and a **radius of 5.00 cm**. This pressure is **transmitted by a liquid** to a piston that has a **radius of 15.0 cm**. What **force must the compressed air** exert to lift a car **weighing 13300 N**? What air **pressure** produces **this force**?

Solution:

$$F_1 = \left(\frac{A_1}{A_2}\right) F_2 = \frac{\pi(5.00 \times 10^{-2} \text{ m})^2}{\pi(15.0 \times 10^{-2} \text{ m})^2}(1.33 \times 10^4 \text{ N})$$
$$= \boxed{1.48 \times 10^3 \text{ N}}$$

The air pressure that produces this force is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3 \text{ N}}{\pi(5.00 \times 10^{-2} \text{ m})^2}$$
$$= \boxed{1.88 \times 10^5 \text{ Pa}}$$

This pressure is approximately twice atmospheric pressure.

Example 7: A **l5 kg** solid **gold** statue is being raised from a sunken ship (Fig. 1-a). What is the **tension** in the hoisting cable when the statue is (a) **at rest and completely immersed**; and (b) **at rest and out of the water**?

If known $\boldsymbol{p_{gold} = 19.3\,\frac{g}{cm^3}}$, $\boldsymbol{p_{swim\ water} = 1.03\,\frac{g}{cm^3}}$, $\boldsymbol{p_{air} = 1.2\,\frac{g}{cm^3}}$.

Solution:

we first find the volume

$$V = \frac{m}{\rho_{gold}} = \frac{15.0\ \text{kg}}{19.3 \times 10^3\ \text{kg/m}^3} = 7.77 \times 10^{-4}\ \text{m}^3$$

we find the weight of this volume of seawater:

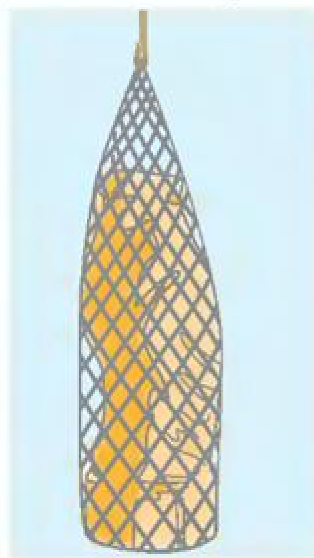
$$w_{sw} = m_{sw}g = \rho_{sw}Vg$$
$$= (1.03 \times 10^3\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2)$$
$$= 7.84\ \text{N}$$

This equals the buoyant force B.

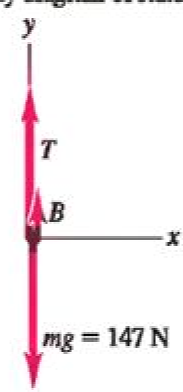
The statue is at rest, so the net external force acting on it is zero. From Fig. 1 b,

$$\sum F_y = B + T + (-mg) = 0$$
$$T = mg - B = (15.0\ \text{kg})(9.80\ \text{m/s}^2) - 7.84\ \text{N}$$
$$= 147\ \text{N} - 7.84\ \text{N} = 139\ \text{N}$$

(a) Immersed statue in equilibrium



(b) Free-body diagram of statue

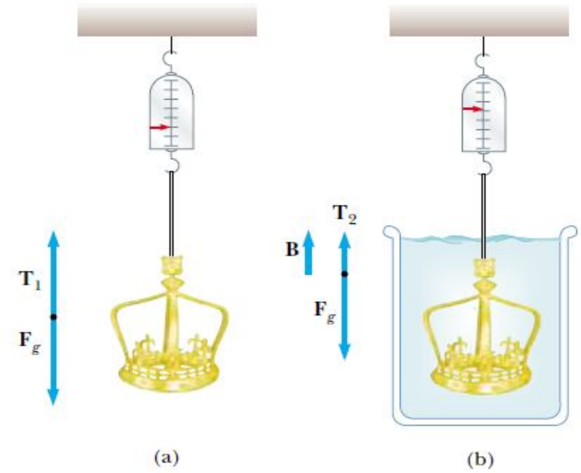


(b) The density of air is about $1.2\ \text{kg/m}^3$, so the buoyant force of air on the statue is

$$B = \rho_{air}Vg = (1.2 \times 10^3\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2)$$
$$= 9.1\ \text{N}$$

Example 8: Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. Legend has it that he solved this problem by weighing the crown first in air and then in water, as shown in this Figure. Suppose the scale read **7.84 *N* in air** and **6.84 *N* in water**. What should Archimedes have told the king?

Solution:



the net force on it is zero. When the crown is in water,

$$\sum F = B + T_2 - F_g = 0$$

so that

$$B = F_g - T_2 = 7.84\ \text{N} - 6.84\ \text{N} = 1.00\ \text{N}$$

$$B = \rho_w g V_w = \rho_w g V_c$$

$$V_c = V_w = \frac{1.00\ \text{N}}{\rho_w g} = \frac{1.00\ \text{N}}{(1\,000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}$$
$$= 1.02 \times 10^{-4}\ \text{m}^3$$

the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{B}{V_c g} = \frac{7.84\ \text{N}}{(1.02 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2)}$$
$$= 7.84 \times 10^3\ \text{kg/m}^3$$

Home Work:

Q 1: A water bed is 2.00 m on a side and 30.0 cm deep. (a) Find its weight. (b) Find the pressure that the water bed exerts on the floor. Assume that the entire lower surface of the bed makes contact with the floor.

Solution:

Q 2: In a huge oil tanker, salt water has flooded an oil tank to a depth of 5.00 m. On top of the water is a layer of oil 8.00 m deep, as in the cross-sectional view of the tank in this Figure. The oil has a density of 0.700 g/cm^3. Find the pressure at the bottom of the tank. (Take 1 025 kg/m^3 as the density of salt water.)

Solution:

Q 3: Estimate the net force exerted on your eardrum due to the water above when you are swimming at the bottom of a pool that is 5.0 m deep.

Solution: