Chapter Four

Surface Tension and Capillary

4.1 Definition and Explanation of Surface Tension:

If you look closely at a dewdrop sparkling in the morning sunlight, you will find that the drop is spherical. The drop takes this shape because of a property of liquid surfaces called **surface tension**. In order to understand the origin of surface tension, consider a molecule at point A in a container of water, as in Figure [1].

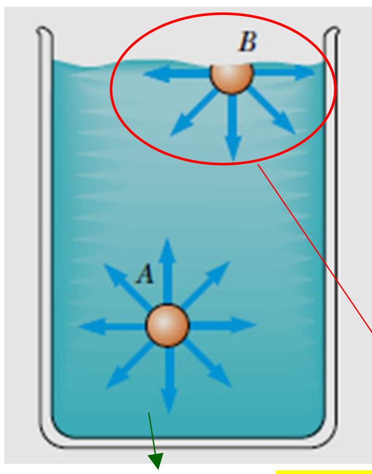

Figure [1]: The **net force on a molecule at A is zero** **because such a molecule is completely surrounded by other molecules**. The **net force on a surface molecule at B is downward** **because it isn't completely surrounded by other molecules**.

Although **nearby molecules** exert **forces on this molecule**, the **net force on it is zero** **because it's completely surrounded by other molecules and hence is attracted equally in all directions**. The **molecule at B**, however, **is not attracted equally in all directions**. **Because there are no molecules above** it to exert upward forces, **the molecule at B is pulled toward the interior of the liquid**. The **contraction at the surface of the liquid ceases (stands)** when the inward pull exerted on the surface molecules is balanced by the outward repulsive forces that arise from collisions with molecules in the interior of the liquid. The net effect of this pulls on all the surface molecules is to make the **surface of the liquid contract** and, consequently, to make the **surface area of the liquid as small as possible**. **Drops of water take on a spherical shape** **because a sphere has the smallest surface area for a given volume**. If you place a sewing needle very carefully on the surface of a bowl of water, you will find that the needle floats even though the density of steel is about eight times that of water. This phenomenon can also be explained by surface tension. A close examination of the needle shows that it actually rests in a depression in the liquid surface as shown in Figure [2].

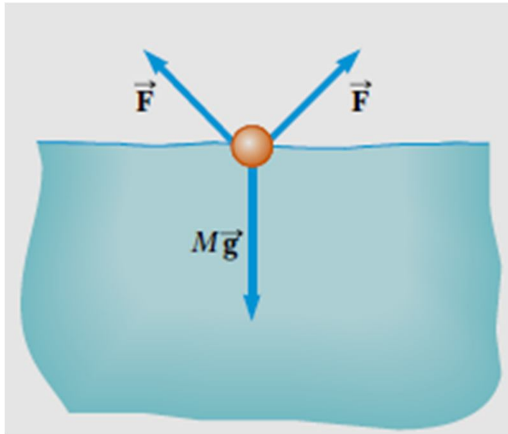

Figure [2]: End view of a needle resting on the surface of water. The components of surface tension balance the force of gravity.

The water surface acts like an **elastic membrane** under tension. The weight of the needle produces a depression, increasing the surface area of the film. Molecular forces now act at all points along the depression, tending to restore the surface to its original horizontal position. The vertical components of these forces act to balance the force of gravity on the needle. The floating needle can be sunk by adding a little detergent to the water, which reduces the surface tension. The **surface tension** (γ) in a film of liquid is defined as **the magnitude of the surface tension force** $(\boldsymbol{F})$ **divided by the length** $(\boldsymbol{L})$ along which the force acts:

$$\gamma = \frac{F}{L} \quad \dots (1)$$

The SI unit of surface tension is the Newton per meter, and values for a few representative materials are given in Table (1). **Surface tension** can be thought of as the **energy content of the fluid at its surface per unit surface area (i.e.,** $\gamma = \frac{E}{A}$**)**. To see that this is reasonable, we can *manipulate the units* of surface tension as follows:

$$\frac{\mathrm{N}}{\mathrm{m}} = \frac{\mathrm{N \cdot m}}{\mathrm{m}^2} = \frac{\mathrm{J}}{\mathrm{m}^2}$$

In general, ***in any equilibrium*** configuration of an object**,** ***the energy is a minimum.*** Consequently, a fluid will take on a shape such that its ***surface area is as small as possible***. For a given volume, ***a spherical shape has the smallest surface area***; *therefore*, ***a drop of water*** takes on ***a spherical shape***.

An apparatus used to measure the **surface tension of liquids** is shown in Figure [3].

TABLE 1

Surface Tensions for Various Liquids

Liquid	T (°C)	Surface Tension (N/m)
Ethyl alcohol	20	0.022
Mercury	20	0.465
Soapy water	20	0.025
Water	20	0.073
Water	100	0.059

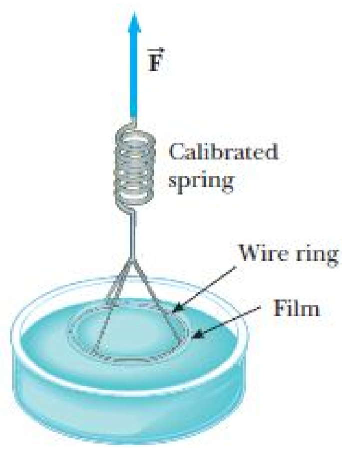

Figure [3]: An apparatus for measuring the surface tension of liquids. The force on the wire ring is measured just before the ring breaks free of the liquid.

A **circular** wire with a **circumference L** is lifted from a body of liquid. The surface film clings to the inside and outside edges of the wire, holding back the wire and causing the spring to stretch. If the spring is calibrated, the force required to overcome the surface tension of the liquid can be measured. *In this case*, **the surface tension is given by**:

$$\gamma = \frac{F}{2L} \quad \dots (2)$$

We use **$2L$ for the length because the surface film exerts (acts) forces on both the inside and outside of the ring**.

The **surface tension of liquids** decreases with increasing **temperature**, **because the faster moving molecules of a hot liquid aren't bound together as strongly as are those in a cooler liquid**.

In addition, certain ingredients called **surfactants** decrease **surface tension** when **added to liquids**. For example, **soap or detergent** **decreases** **the surface tension of water**, making it easier for soapy water to penetrate the cracks and crevices of your clothes to clean them **better than** plain water does.

A similar effect occurs in the **lungs**. The surface tissue of the air sacs in the **lungs** contains **a fluid** that has a **surface tension of about $0.05\ N/m$**. A liquid with a surface tension this **high** would make it **very difficult** for the **lungs to expand during inhalation**. However, as the **area of the lungs increases** with inhalation, the body secretes into the tissue a substance that gradually **reduces the surface tension** of the liquid.
At **full expansion**, the **surface tension of the lung** fluid can drop to as low as $\mathbf{0.005\ N/m}$.

4.2 Shape of Liquid Meniscus in a Glass Tube:

If you have ever closely examined the surface of water in a glass container, you may have noticed that the surface of the liquid near the walls of the glass curves upwards as you move from the center to the edge, as shown in Figure 4a. However, if mercury is placed in a glass container, the mercury surface curves downwards, as in Figure 4-b. These surface effects can be explained by considering the forces between molecules. In particular, we must consider the forces that the molecules of the liquid exert on one another and the forces that the molecules of the glass surface exert on those of the liquid. In general terms, **forces between like molecules**, **such as the forces between water molecules**, are called **cohesive forces**, and **forces between unlike molecules**, **such as those exerted by glass on water**, are called **adhesive forces**.

Water tends to cling to the walls of the glass because the adhesive forces between the molecules of water and the glass molecules **are greater than the cohesive forces** between the water molecules. In effect, the water molecules cling to the surface of the glass rather than fall back into the bulk of the liquid. When this condition prevails, the **liquid is said to "wet" the glass surface**.
The surface of the mercury curves downward near the walls of the container because **the cohesive forces** between the mercury atoms **are greater than the adhesive forces** between mercury and glass. A mercury atom near the surface is pulled more strongly toward other mercury atoms than toward the glass surface, **so mercury doesn't wet the glass surface**.

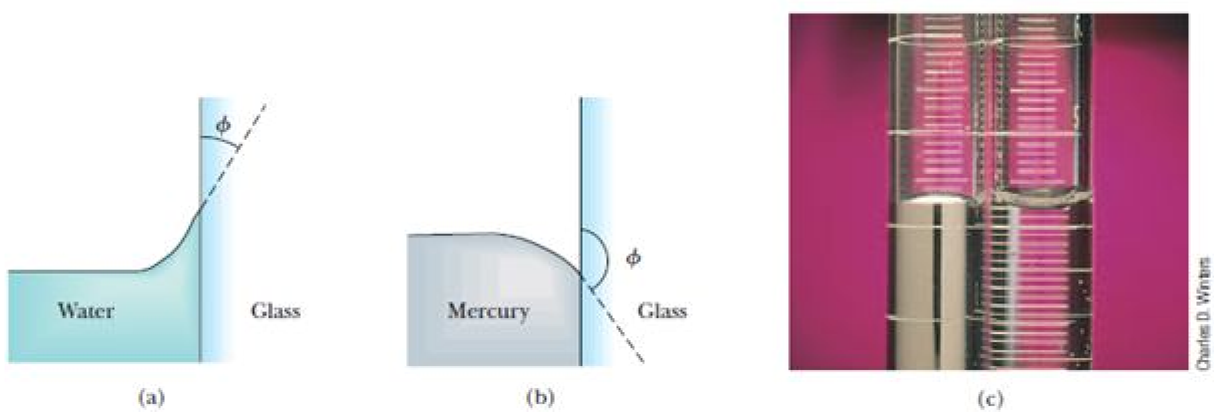

Figure [4]: A liquid in contact with a solid surface. (a) For water, the adhesive force is greater than the cohesive force. (b) For mercury, the adhesive force is less than the cohesive force. (c) The surface of mercury (left) curves downwards in a glass container, whereas the surface of water (right) curves upwards, as you move from the center to the edge.

4.3 Capillary Action:

$$\boxed{L \cong 100\, r}$$

In capillary tubes **the diameter of the opening is very small**, on the order of **a hundredth of a centimeter**. In fact, the word **capillary** means “**hair like**”.

If such a tube is inserted into a fluid for which adhesive forces dominate over cohesive forces, the *liquid rises into the tube*, as shown in Figure 5. The rising of the liquid in the tube can be explained in terms of the shape of the liquid’s surface and surface tension effects.

At the point of contact between liquid and solid, the upward force of surface tension is directed as shown in the figure. **The magnitude of this force is**:

$$F = \gamma L = \gamma(2\pi r) \quad \ldots.(3)$$

(We use $L = 2\pi r$ here because the liquid is in contact with the surface of the tube at all points around its **circumference**.)

The vertical component of this force due to surface tension is:

$$F_v = \gamma(2\pi r)\,(\cos \emptyset) \quad \ldots(4)$$

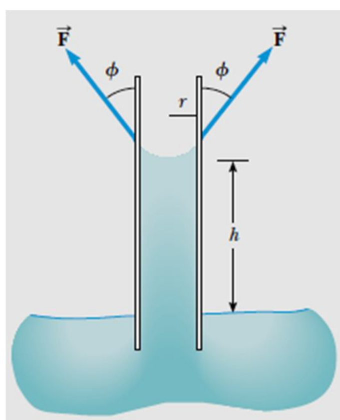

Figure [5]: A liquid rises in a narrow tube because of capillary action, a result of surface tension and adhesive forces.

In order for the liquid in the capillary tube to be in equilibrium, this upward force must be equal to the weight of the cylinder of water of height h inside the capillary tube. The weight of this water is:

$$W = mg = \rho V g = \rho g \pi r^2 h \quad \ldots(5)$$

Equating F_v (**applying Newton's second law for equilibrium**), we have:

$$F_v = W$$

$$\gamma(2\cancel{\pi}\cancel{r})(\cos\emptyset) = \rho g \cancel{\pi} r^{\cancel{2}} h$$

Solving for ***h* gives the height** to which water is drawn into the tube:

$$h = \frac{2\gamma}{\rho g r}\cos\emptyset \quad \ldots (6)$$

If a capillary tube is inserted into a liquid in which cohesive forces dominate over adhesive forces, the level of the liquid in the capillary tube will be below the surface of the surrounding fluid, as shown in Figure 6. An analysis similar to the above would show that the **height *h*** to the depressed surface is given by the **last equation**.

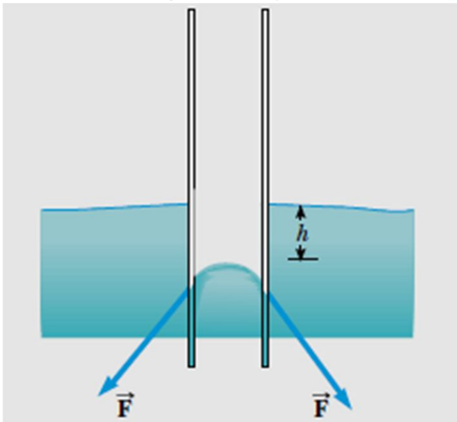

Figure [6]: When cohesive forces between molecules of a liquid exceed adhesive forces, the level of the liquid in the capillary tube is below the surface of the surrounding fluid.

- **Capillary tubes** are often **used** to draw **small samples of blood from a needle prick in the skin**.
- **Capillary action** must also be considered in the **construction of concrete-block buildings**, because water seepage through capillary pores in the blocks or the mortar may cause damage to the inside of the building. To prevent such damage, the blocks are usually coated with a waterproofing agent either outside or inside the building. Water seepage through a wall is an undesirable effect of capillary action, but there are many useful effects.
- **Plants** depend on **capillary action** to transport water and nutrients, and
- **sponges** and **paper towels** use **capillary action** to absorb spilled fluids.

Example 1: Many insects can literally walk on **water**, using surface tension for their support. To show this is feasible, assume that the insect's "foot" is **spherical**. When the insect steps onto the water with all six legs, a depression is formed in the water around each foot, as shown in Figure (1-a). The surface tension of the water produces upward forces on the water that tend to restore the water surface to its normally flat shape. If the insect has a **mass of $2.0 \times 10^{-5}\ kg$** and if the **radius of each foot is $1.5\ \times 10^{-4}\ m$**, find the **angle θ**. (If known **surface tension** of water is **$0.073\ N/m$**).

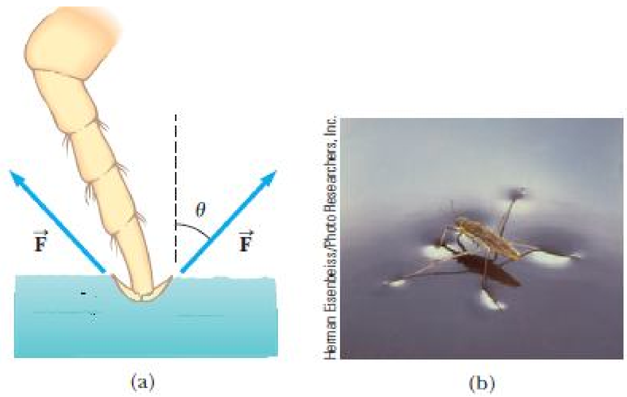

Figure [1]: (a) One foot of an insect resting on the surface of water. (b) This water strider resting on the surface of a lake remains on the surface, rather than sinking, because an upward surface tension force acts on each leg, balancing the force of gravity on the insect.

Solution:

$$F_v = \gamma(2\pi r)\,(\cos\emptyset)$$

$$\sum F = F_v - F_{grav} = 0$$

$$\gamma(2\pi r)\,(\cos\emptyset) - \frac{1}{6}mg = 0 \;\Rightarrow\; \gamma(2\pi r)\,(\cos\emptyset) = \frac{1}{6}mg \;\Rightarrow\; \cos\emptyset = \frac{mg}{12\pi r\gamma}$$

$$\cos\emptyset = \frac{2\times 10^{-5}\times 9.8}{12\pi\times 1.5\ \times 10^{-4}\times 0.073} = 0.47$$

$$\emptyset = \cos^{-1} 0.47 = 62^{\circ}$$

Example 2: Find the **height** to which **water** would rise in a capillary tube with a **radius equal to $5\times\ 10^{-5}m$**. Assume that the contact **angle** between the water and the material of the tube is small enough to be considered **zero**. (If known **surface tension** of water is **$0.073\ N/m$**).

Solution:

$$h = \frac{2\gamma}{\rho g r}\cos\emptyset$$

$$h = \frac{2\gamma}{\rho g r}\cos 0 = \frac{2\times 0.073}{1\times 10^{3}\times 9.8\times 5\times 10^{-5}} = 0.3\ m$$

Example 3: A certain fluid has a **density of 1080 kg/m^3** and is observed to rise to a **height of 2.1 cm** in a **1.0 mm diameter** tube. The contact **angle between the wall and the fluid is zero**. Calculate the **surface tension** of the fluid.

Solution:

$$h = \frac{2\gamma}{\rho g r} \cos \emptyset$$

$$\gamma = \frac{h\rho g r}{2 \cos \emptyset} = \frac{2.1 \times 10^{-2} \times 1080 \times 9.8 \times 0.5 \times 10^{-3}}{2 \times \cos 0} = 0.056 \frac{N}{m}$$

Home Work:

Q 1: In order to lift a wire ring of radius 1.75 cm from the surface of a container of blood plasma, a vertical force of 1.61×10^{-2} N greater than the weight of the ring is required. Calculate the surface tension of blood plasma from this information.

Solution:

Q 2: Whole blood has a surface tension of 0.058 N/m and a density of 1050 kg/m^3. To what height can whole blood rise in a capillary blood vessel that has a radius of 2.0×10^{-6} m if the contact angle is zero?

Solution:

Q 3: Suppose ethyl alcohol rises 0.250 m in a thin tube. Estimate the radius of the tube, assuming the contact angle is approximately zero. Assume that density is 805 kg/m^3 and $\gamma = 0.022$.

Solution: