## Chapter Five <br> Viscosity

### 5.1 Viscosity of Liquid:

Whenever a liquid flows on a horizontal surface, the velocities of the different layers of the liquid parallel to the fixed surface are different and increase with the distance from the fixed surface. This property is called viscosity or internal friction.
In a general sense, viscosity refers to the internal friction of a fluid.
The force $(\boldsymbol{F})$ required moving the upper plate at a fixed speed $(\boldsymbol{v})$ is therefore:

$$
\begin{aligned}
F & =-\eta A \frac{d v}{d x} \\
\text { Or } F & =\eta A \frac{v}{d} \ldots(1)
\end{aligned}
$$

where $\boldsymbol{\eta}$ is the coefficient of viscosity of the fluid, $\frac{d v}{d x}$ is velocity gradient (change of velocity with distance), and $\boldsymbol{A}$ is area.
The negative sign shows that the force is acting opposite to the direction of velocity. The SI units of viscosity are $\frac{N \cdot s}{m^{2}}$. The units of viscosity in many reference sources are often expressed in $\frac{\text { dyne. } s}{{c m^{2}}^{2}}$, called 1 poise, in honor of the French scientist J. L. Poiseuille (1799-1869). The relationship between the SI unit of viscosity and the poise is:

$$
1 \text { Poise }=10^{-1} \frac{N . s}{m^{2}}
$$

Small viscosities are often expressed in centipoise (cp), where $1 \mathbf{c p}=10^{-2}$ poise. The coefficients of viscosity for some common substances are listed in Table [1].

TABLE 1

| Viscosities of Various Fluids |  |  |
| :--- | :---: | :---: |
| Fluid | $\boldsymbol{T}\left({ }^{\circ} \mathbf{C}\right)$ | Viscosity $\boldsymbol{\eta}\left(\mathbf{N} \cdot \mathbf{s} / \mathbf{m}^{2}\right)$ |
| Water | 20 | $1.0 \times 10^{-3}$ |
| Water | 100 | $0.3 \times 10^{-3}$ |
| Whole blood | 37 | $2.7 \times 10^{-3}$ |
| Glycerin | 20 | $1500 \times 10^{-3}$ |
| 10 -wt motor oil | 30 | $250 \times 10^{-3}$ |

Poise: The coefficient of viscosity of a liquid is poise, if a force of 1 Dyne is required to maintain a velocity gradient of one unit between two layers of area $1 \mathbf{c m}^{2}$ each.

### 5.2 Viscosity of Gases:

The coefficient of viscosity of gases can be determined by using liquid equations, with certain modification.
Liquids are practically incompressible and hence the density of the liquid almost remains constant irrespectity (بَّ گويّان) of the changes in pressure. But in the case of gases the density is varies with pressure.
For liquids, the volume (or mass) flowing per second across any cross-section is constant. In the case of gases, the mass (not volume) of the gas flowing per second at any crosssection is constant.

### 5.3 Poiseuille's Method for Coefficient of Viscosity:

Consider a liquid flowing in a capillary tube:
$L$ : Length of tube, $r$ : Radius of tube and
$v$ : Velocity at all points on the cylindrical shell of radius
The tangential force acting in opposite direction to direction of flow is given by:

$$
F=-\eta A \frac{d v}{d x}
$$

where $A=\mathbf{2 \pi x L}$, Then above equation becomes:

$$
F=-2 \pi \eta x L \frac{d v}{d x}
$$

But $F=P A=P \pi x^{2}$
where $P=P_{1}-P_{2}$ is the difference in pressure between the two ends of the capillary tube.
From these two equations:

$$
\begin{gathered}
P \pi x^{2}=-2 \pi \eta x L \frac{d v}{d x} \\
d v=-\frac{\hbar \tau P x^{k}}{2 \not \hbar \eta \nmid \chi L} d x \ldots \text { (2) }
\end{gathered}
$$

We take integrating for both sides in equation (2), we get;

$$
\begin{gathered}
\int d v=-\frac{P}{2 \eta L} \int x d x \rightarrow v=-\frac{P}{2 \eta L} \cdot \frac{x^{2}}{2}+C \\
v=-\frac{\boldsymbol{P} \boldsymbol{x}^{2}}{4 \boldsymbol{\eta} \boldsymbol{L}}+\boldsymbol{C} \ldots(3)
\end{gathered}
$$

Now $\boldsymbol{v}=\mathbf{0}$ when $\boldsymbol{x}=\boldsymbol{r}$

$$
\therefore 0=-\frac{P r^{2}}{4 \eta L}+C \Rightarrow C=\frac{P r^{2}}{4 \eta L}
$$

Substituting $\mathbf{C}$ in equation (3):

$$
\begin{equation*}
v=\frac{P}{4 \eta L}\left(r^{2}-x^{2}\right) \ldots \tag{4}
\end{equation*}
$$

This is the velocity of flow of the liquid at a distance $\boldsymbol{x}$ from the axis of the tube.

Volume of the liquid between the two layers per second:

$$
\begin{equation*}
\varphi=\frac{d V}{d t}=v A \rightarrow \boldsymbol{d} \varphi=v \boldsymbol{d} A \ldots \tag{5}
\end{equation*}
$$

where $\boldsymbol{A}=\boldsymbol{\pi} \boldsymbol{x}^{\mathbf{2}} \rightarrow \boldsymbol{d} \boldsymbol{A}=\mathbf{2} \boldsymbol{\pi} \boldsymbol{x} \boldsymbol{d} \boldsymbol{x}$, and substituting equation (4) into equation (5), then equation (5) becomes:

$$
\begin{equation*}
d \varphi=\left(\frac{P}{\not 2 * 2 \eta L}\left(r^{2}-x^{2}\right)\right)(\not 2 \pi x d x) \ldots \tag{6}
\end{equation*}
$$

Rate of flow of the liquid through the whole tube is given by:

$$
\begin{gather*}
\varphi=\frac{\pi P}{2 \eta L} \int_{0}^{r}\left(r^{2}-x^{2}\right) x d x \\
\varphi=\frac{\pi P}{2 \eta L}\left[\frac{r^{2} x^{2}}{2}-\frac{x^{4}}{4}\right]_{0}^{r}=\frac{\pi P}{2 \eta L}\left[\frac{r^{2} r^{2}}{2}-\frac{r^{4}}{4}\right]=\frac{\pi P}{2 \eta L}\left(\frac{r^{4}}{4}\right) \\
\varphi=\frac{\pi P r^{4}}{8 \eta L} \ldots(7) \tag{7}
\end{gather*}
$$

and

$$
\eta=\frac{\pi P r^{4}}{8 \varphi L} \ldots \text { (7) }
$$

where $\boldsymbol{\eta}$ is the coefficient of viscosity of the fluid.
Equation (7) represent of Poiseuille's Law.

### 5.4 Reynolds Number:

At sufficiently high velocities, fluid flow changes from simple streamline flow to turbulent flow, characterized by a highly irregular motion of the fluid.
Experimentally, the onset of turbulence in a tube is determined by a dimensionless factor called the Reynolds number (RN), given by:

$$
R N=\frac{\rho v d}{\eta} \ldots(8)
$$

where $\boldsymbol{\rho}$ is the density of the fluid, $\boldsymbol{v}$ is the average speed of the fluid along the direction of flow, $\boldsymbol{d}$ is the diameter of the tube, and is the viscosity of the fluid. If RN is below about 2000, the flow of fluid through a tube is streamline; turbulence occurs if RN is above 3000. In the region between 2000 and 3000, the flow is unstable, meaning that the fluid can move in streamline flow, but any small disturbance will cause its motion to change to turbulent flow.

Example 1: A flat plate of area $\mathbf{1 0} \mathbf{~ c m}^{2}$ is separated from a large plate by a layer of glycerin 1 mm thick. If the viscous coefficient of glycerin is $20 \mathrm{~g} / \mathbf{c m} \cdot \boldsymbol{s e c}$, what force is required to keep the plate moving with a velocity of $1 \mathrm{~cm} / \mathbf{s e c}$ ?

## Solution:

$\eta=20 \mathrm{~g} / \mathrm{cm} \cdot \mathrm{sec}, A=10 \mathrm{~cm}^{2}, v=1 \frac{\mathrm{~cm}}{\mathrm{sec}}, x=1 \mathrm{~mm}=0.1 \mathrm{~cm}$
$F=-\boldsymbol{\eta} A \frac{d v}{d x}$
$F=-20 \times 10 \times \frac{1}{0.1}=2000$ dynes
Example 2: A patient receives a blood transfusion through a needle of radius $\mathbf{0 . 2 0} \mathbf{~ m m}$ and length 2.0 cm . The density of blood is $1050 \mathrm{~kg} / \mathbf{m}^{3}$. The bottle supplying the blood is 0.50 m above the patient's arm. What is the rate of flow through the needle?

$$
\left(\eta=0.0027 N . s / m^{2}\right)
$$

## Solution:

$$
\begin{aligned}
& \boldsymbol{P}=\boldsymbol{P}_{\mathbf{1}}-\boldsymbol{P}_{2}=\boldsymbol{\rho} \boldsymbol{g} \boldsymbol{h}=1050 \times 9.8 \times 0.5=5.15 \times 10^{3} \mathrm{~Pa} \\
& \boldsymbol{\varphi}=\frac{\boldsymbol{\pi} \boldsymbol{P r}^{4}}{\boldsymbol{8} \boldsymbol{\eta} \boldsymbol{L}}=\frac{\pi \times 2 \times 10^{-4} \times 5.15 \times 10^{3}}{8 \times 2.7 \times 10^{-3} \times 2 \times 10^{-2}}=6 \times 10^{-8} \boldsymbol{m}^{3} / \boldsymbol{s}
\end{aligned}
$$

Example 3: Determine the speed at which blood flowing through an artery of diameter $\mathbf{0 . 2 0} \mathbf{c m}$ will become turbulent. (In example 3). $\mathbf{R N}=\mathbf{3 0 0 0}$.

## Solution:

$$
\begin{aligned}
& v=\frac{\eta(R N)}{\rho d}=\frac{\left(2.7 \times 10^{-3} \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}^{2}\right)\left(3.00 \times 10^{3}\right)}{\left(1.05 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.20 \times 10^{-2} \mathrm{~m}\right)} \\
& v=3.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Example 4: Water flows through a horizontal capillary tube of 1 mm internal diameter and length 70 cm under pressure of a column of water 30 cm in height. Find the rate of water through the capillary tube. Viscosity of water $=\mathbf{2 0}^{\mathbf{- 3}} \mathrm{N} . \boldsymbol{S} / \boldsymbol{m}^{\mathbf{2}}$.

## Solution:

$$
\begin{aligned}
& r=0.5 \mathrm{~mm}=5 \times 10^{-4} \mathrm{~m}, L=0.7 \mathrm{~m}, \quad \eta=10^{-3} \mathrm{~N} . \mathrm{s} / \mathrm{m}^{2} \\
& P=30 \mathrm{~cm}=0.3 \mathrm{~m} \text { of water }=0.3 \times 10^{3} \times 9.8 \mathrm{~N} / \mathrm{m}^{2} \\
& \boldsymbol{\varphi}=\frac{\boldsymbol{\pi} \boldsymbol{P r}}{\mathbf{4} \boldsymbol{\eta} \boldsymbol{L}} \\
& \varphi=\frac{\pi \times 0.3 \times 10^{3} \times 9.8 \times\left(5 \times 10^{-4}\right)^{4}}{8 \times 10^{-3} \times 0.7} \\
& \varphi=1.038 \times 10^{-7} \mathrm{~m}^{3} / \mathrm{s}
\end{aligned}
$$

## Home Work:

Q 1: A pipe carrying water from a tank 20 m tall must cross $3 \times 10^{2} \mathrm{~km}$ of wilderness to reach a remote town. Find the radius of pipe so that the volume flow rate is at least $0.05 \mathrm{~m}^{3} / \mathrm{s}$. (Use the viscosity of water at $20^{\circ} \mathrm{C}$.)
Answer: 0.118 m

Q 2: Determine the speed v at which water at $20^{\circ}$ sucked up a straw would become turbulent. The straw has a diameter of 0.0060 m .
Answer: $\mathbf{0 . 5 0} \mathbf{~ m} / \mathrm{s}$

