Chapter Six

Transport Phenomena

6.1 Introduction:

When a fluid flows through a tube, the basic mechanism that produces the flow is a difference in pressure across the ends of the tube. This pressure difference is responsible for the transport of a mass of fluid from one location to another. The fluid may also move from place to place because of a second mechanism—one that depends on a difference in concentration between two points in the fluid, as opposed to a pressure difference. When the **concentration** (the **number of molecules per unit volume**) is higher at one location than at another, molecules will flow from the point where the concentration is high to the point where it is lower. The two fundamental processes involved in fluid transport resulting from concentration differences are called **diffusion and osmosis**.

6.2 Diffusion:

In a diffusion process, molecules move from a region where their concentration is high to a region where their concentration is lower.

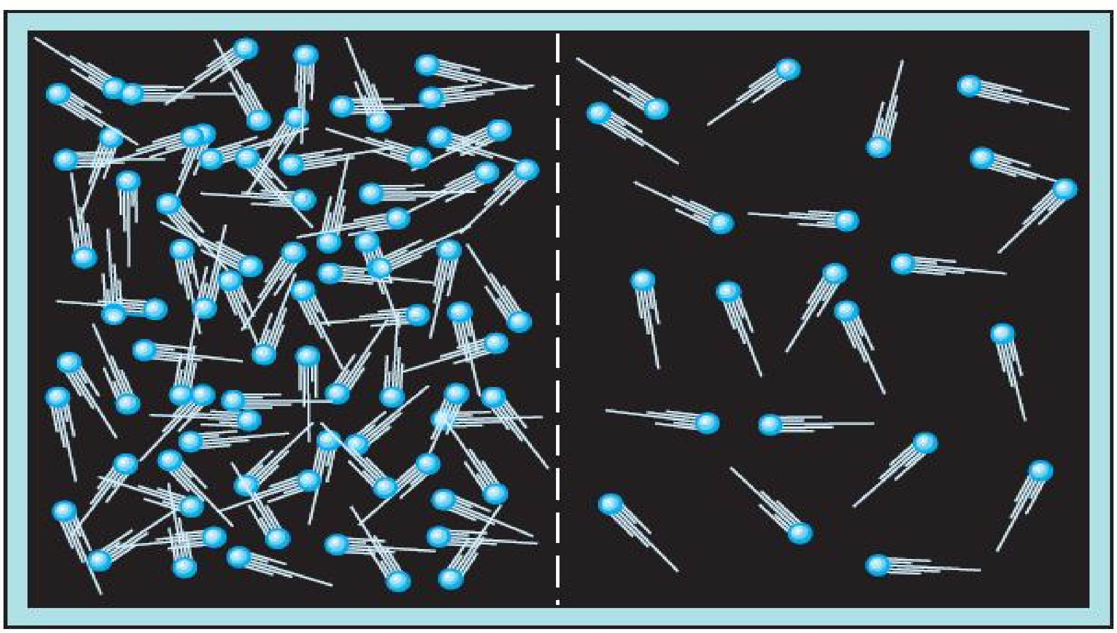

Figure [1]: When the concentration of gas molecules on the left side of the container exceeds the concentration on the right side, there will be a net motion (diffusion) of molecules from left to right.

To understand why diffusion occurs, consider Figure [1], which depicts a container in which a high concentration of molecules has been introduced into the left side. The dashed line in the figure represents an imaginary barrier separating the two regions. Because the molecules are moving with high speeds in random directions, many of them will cross the imaginary barrier moving from left to right. Very few molecules will pass through moving from right to left, simply because there are very few of them on the right side of the container at any instant. As a result, there will always be a net movement from the region with many molecules to the region with fewer molecules. For this reason, the concentration on the left side of the container will decrease, and that on the right side will increase with time. Once concentration equilibrium has been reached, there will be no net movement across the cross-sectional area: The rate of movement of molecules from left to right will equal the rate from right to left.

The basic equation for diffusion is **Fick's law**,

$$\boldsymbol{Diffusion\,rate = \frac{mass}{time} = \frac{dm}{dt} = DA\left(\frac{C_2 - C_1}{L}\right)} \quad \dots(\mathbf{1})$$

where **D is a constant of proportionality**. The left side of this equation is called the diffusion rate and is a measure of the mass being transported per unit time. The equation says that the rate of diffusion is proportional to the **cross-sectional area A** and to the **change in concentration per unit distance,** $(C_2 - C_1)/L$, which is called the concentration gradient. The concentrations **C_1 and C_2 are measured in kilograms per cubic meter**. **The proportionality constant D is called the diffusion coefficient and has units of square meters per second**.

Table (1) lists diffusion coefficients for a few substances.

TABLE 1

Diffusion Coefficients of Various Substances at 20°C

Substance	D (m^2/s)
Oxygen through air	6.4×10^{-5}
Oxygen through tissue	1×10^{-11}
Oxygen through water	1×10^{-9}
Sucrose through water	5×10^{-10}
Hemoglobin through water	76×10^{-11}

6.3: The Size of Cells and Osmosis:

Diffusion through cell membranes is vital in carrying oxygen to the cells of the body and in removing carbon dioxide and other waste products from them. Cells require oxygen for those metabolic processes in which substances are either synthesized or broken down. In such processes, the cell uses up oxygen and produces carbon dioxide as a by-product. A fresh supply of oxygen diffuses from the blood, where its concentration is high, into the cell, where its concentration is low. Likewise, carbon dioxide diffuses from the cell into the blood, where it is in lower concentration. Water, ions, and other nutrients also pass into and out of cells by diffusion.

A cell can function properly only if it can transport nutrients and waste products rapidly across the cell membrane. The surface area of the cell should be large enough so that the exposed membrane area can exchange materials effectively while the volume should be small enough so that materials can reach or leave particular locations rapidly. This requires a large surface-area-to-volume ratio.

Model a cell as a cube, each side with length L. **The total surface area is $6L^2$** and **the volume is L^3**. The surface area to volume is then;

$$\frac{\text{surface area}}{\text{volume}} = \frac{6L^2}{L^3} = \frac{6}{L}$$

Because **L is in the denominator**, **a smaller L means a larger ratio**. This shows that the smaller the size of a body, the more efficiently it can transport nutrients and waste products across the cell membrane. Cells range in size from a millionth of a meter to several millionths, so a good estimate of a typical cell's surface-to-volume ratio is 10^6.

The diffusion of material through a membrane is partially determined by the size of the pores (holes) in the membrane wall. Small molecules, such as water, may pass through the pores easily, while larger molecules, such as sugar, may pass through only with difficulty or not at all. A membrane that allows passage of some molecules but not others is called a selectively permeable membrane.

Osmosis is the diffusion of water across a selectively permeable membrane from a high-water concentration to a low water concentration. As in the case of diffusion, osmosis continues until the concentrations on the two sides of the membrane are equal.

6.4 Motion through a Viscous Medium:

When an object falls through air, its motion is impeded by the force of air resistance. In general, this force is dependent on the shape of the falling object and on its velocity. The force of air resistance acts on all falling objects, but the exact details of the motion can be calculated only for a few cases in which the object has a simple shape, such as a sphere. In this section, we will examine the motion of a tiny spherical object falling slowly through a viscous medium.

In 1845 a scientist named George Stokes found that the magnitude of the resistive force on a very small spherical object of radius r falling slowly through a fluid of viscosity ν with speed v is given by:

$$\boldsymbol{F_r = 6\,\pi\,\eta\,r\,v} \quad ...(2)$$

This equation, called Stokes's law, has many important applications. For example, it describes the sedimentation of particulate matter in blood samples. It was used by Robert Millikan (1886–1953) to calculate the radius of charged oil droplets falling through air. From this, Millikan was ultimately able to determine the charge on the electron, and was awarded the Nobel Prize in 1923 for his pioneering work on elemental charges.

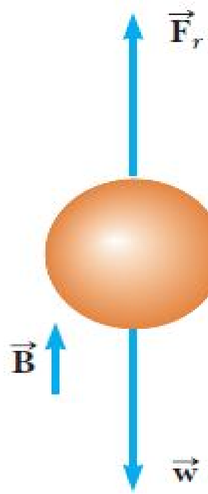

Figure [2]: A sphere falling through a viscous medium. The forces acting on the sphere are the resistive frictional force $\overrightarrow{F_r}$, the buoyant force $\vec{B}$, and the force of gravity acting on the sphere.

As a sphere falls through a viscous medium, three forces act on it, as shown in Figure [2]: **$\overrightarrow{F_r}$, the force of friction**; **$\vec{B}$, the buoyant force of the fluid**; and **$\vec{w}$, the force of gravity acting on the sphere**. The magnitude of is given by:

$$w = \rho g V = \rho g \left(\frac{4}{3}\pi r^3\right) \quad ...(3)$$

where ρ **is the density of the sphere** and is $\frac{4}{3}\pi r^3$ **its volume**. According to Archimedes's principle, the magnitude of the buoyant force is equal to the weight of the fluid displaced by the sphere:

$$B = \rho_f g V = \rho_f g\left(\frac{4}{3}\pi r^3\right) \quad \ldots (4)$$

where ρ_f **is the density of the fluid.**

At the instant the sphere begins to fall, the force of friction is zero because the speed of the sphere is zero. As the sphere accelerates, its speed increases and so does $\overrightarrow{F_r}$. **Finally, at a speed called the terminal speed v_t , the net force goes to zero**. This occurs when the net upward force balances the downward force of gravity. Therefore, the sphere reaches terminal speed when:

$$F_r + B = w$$

Or

$$6\pi\eta r v_t + \rho_f g\left(\frac{4}{3}\pi r^3\right) = \rho g\left(\frac{4}{3}\pi r^3\right) \quad \ldots (5)$$

When this equation is solved for v_t, we get:

$$v_t = \frac{2r^2 g}{9\,\eta}(\rho - \rho_f) \quad \ldots (6)$$

6.5 Sedimentation and Centrifugation:

If an object isn't spherical, we can still use the basic approach just described to determine its terminal speed. The only difference is that we can't use Stokes's law for the resistive force. Instead, we assume that the **resistive force has a magnitude given by $F_r = kv$**, where **k is a coefficient that must be determined experimentally**. As discussed previously, the object reaches its terminal speed when the downward force of gravity is balanced by the net upward force, or

$$w = B + F_r \quad \ldots (7)$$

where $B = \rho_f\, g\, V$ **is the buoyant force**. The **volume V** of the displaced fluid is related to the **density** ρ of the falling object by $V = m/\rho$. Hence, we can express the buoyant force as:

$$B = \frac{\rho_f}{\rho} mg \quad \ldots (8)$$

We substitute this expression for B and $\boldsymbol{F_r} = \boldsymbol{k v_t}$ into Equation (7) (terminal speed condition):

$$mg = \frac{\rho_f}{\rho} mg + kv_t$$

or

$$v_t = \frac{mg}{k}\left(1 - \frac{\rho_f}{\rho}\right) \quad \textbf{... (9)}$$

The terminal speed for particles in biological samples is usually quite small. For example, the terminal speed for blood cells falling through plasma is about **5 cm/h** in the gravitational field of the Earth. The terminal speeds for the molecules that make up a cell are many orders of magnitude smaller than this because of their much smaller mass. The speed at which materials fall through a fluid is called the **sedimentation rate** and is important in clinical analysis.

The sedimentation rate in a fluid can be increased by increasing the effective acceleration g that appears in Equation (9). A fluid containing various biological molecules is placed in a centrifuge and whirled at very high angular speeds (Fig. 3). Under these conditions, the particles gain a large radial acceleration $a_c = v^2/r = \omega^2 r$ that is much greater than the free-fall acceleration, so we can replace g in Equation (9) by $\omega^2 r$ and obtain:

$$v_t = \frac{m\omega^2 r}{k}\left(1 - \frac{\rho_f}{\rho}\right) \quad \textbf{... (10)}$$

This equation indicates that the sedimentation rate is enormously speeded up in a centrifuge ($\omega^2 r \gg g$) and that those particles with the greatest mass will have the largest terminal speed. Consequently, the most massive particles will settle out on the bottom of a test tube first.

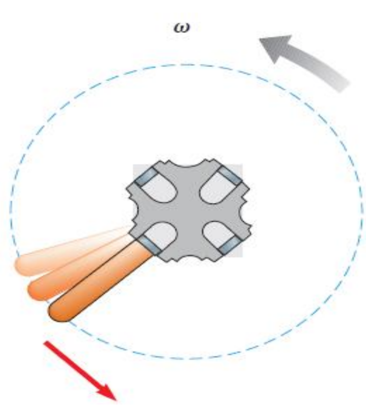

Figure [3]: Simplified diagram of a centrifuge (top view).

Example 1: Sucrose is allowed to diffuse along a **10 cm length** of tubing filled with **water**. The tube is **6.0 cm^2 in cross-sectional area**. The **diffusion coefficient is equal to** $\mathbf{5.0 \times 10^{-10}\, m^2/s}$, and $\mathbf{8.0 \times 10^{-14}\, kg}$ is transported along the tube in **15 s**. **What is the difference in the concentration levels of sucrose at the two ends of the tube**?

$$Diffusion\ rate = \frac{mass}{time} = DA\left(\frac{C_2 - C_1}{L}\right)$$

$$\frac{8 \times 10^{-14}\, kg}{15\, s} = \left(5 \times 10^{-10}\frac{m^2}{s}\right)(6 \times 10^{-4} m^2)\left(\frac{C_2 - C_1}{0.1\, m}\right)$$

$$C_2 - C_1 = \frac{8 \times 10^{-14} \times 0.1}{15 \times 5 \times 10^{-10} \times 6 \times 10^{-4}} = 0.00178\frac{kg}{m^3}$$

Example 2: The viscous force on an oil drop is measured to be $\mathbf{3.0 \times 10^{-13}\, N}$ when the drop is falling through air with a speed of $\mathbf{4.5 \times 10^{-4}\, m/s}$. If the radius of the drop is $\mathbf{2.5 \times 10^{-6}\, m}$, what is the viscosity of air?

$$F = \rho\, g\, V = \rho\, g\frac{4}{3}\pi\, r^3$$

$$3 \times 10^{-13}\, N = \rho\, (9.8\, m/s^2)\frac{4}{3}\pi\, (2.5 \times 10^{-6} m)^3$$

$$\rho = \frac{3 \times 10^{-13}\, N \times 3}{4 \times (9.8\, m/s^2) \times \pi \times (2.5 \times 10^{-6} m)^3} = 467.72\ \frac{kg}{m^3}$$

$$v_t = \frac{2r^2 g}{9\, \eta}(\rho - \rho_f)$$

$$\eta = \frac{2(2.5 \times 10^{-6} m)^2 (9.8\, m/s^2)}{9\, (4.5 \times 10^{-4})}(467.72 - 1.2)\, kg/m^3$$

$$\eta = 1.4 \times 10^{-5}\ \frac{N.s}{m^2}$$

Example 3: Calculate the terminal velocity of an air bubble of radius 0.5mm rising in a liquid of viscosity $0.3Ns/m^2$. Density of the liquid $= 900Kg/m^3$. (Neglect density of air in comparison to that of the liquid.

$$\eta = \frac{2r^2(\rho - \rho_f)g}{9v}$$

$$v = \frac{2r^2(\rho - \rho_f)g}{9\eta}$$

$$v = \frac{2 \times (0.5 \times 10^{-3})^2 \times 9.8 \times 900}{9 \times 0.3} = 1.63 \times 10^{-3}\ m/s$$

Home Work:

Q 1: Glycerin in water diffuses along a horizontal column that has a cross-sectional area of $\mathbf{2.0\ cm^2}$. The concentration gradient is $\mathbf{3.0 \times 10^{-2}\ kg/m^4}$, and the diffusion rate is found to be $\mathbf{5.7 \times 10^{-15}\ kg/s}$. **Determine the diffusion coefficient**.

Solution:

$$\boldsymbol{Diffusion\ rate = \frac{mass}{time} = DA\left(\frac{C_2 - C_1}{L}\right)}$$

$$\boldsymbol{5.7 \times 10^{-15}\frac{kg}{s} = D(2 \times 10^{-4} m^2)\left(3 \times 10^{-2}\frac{kg}{m^4}\right)}$$

$$\boldsymbol{D = \frac{5.7 \times 10^{-15}}{3 \times 10^{-2} \times 2 \times 10^{-4}} = 0.95 \times 10^{-9}\frac{m^2}{s}}$$

Q 2: Two equal drops of water each of radius r, are falling through air, with a steady **velocity v**. If the two drops coalesce to form a bigger drop, **find the new velocity of fall**.

Solution:

Let the radius of the bigger drop be R.

$$V_R = \frac{4}{3}\pi R^3 = 2V_r = 2\left(\frac{4}{3}\pi r^3\right)$$

$$R = (2)^{1/3}\, r = 1.26\, r$$

$$\eta = \frac{2r^2(\rho - \rho_f)g}{9v}$$

For water $\rho = 1g/cm^3$

Density of air was neglected comparing to water.

Small drop : $\eta = \dfrac{2r^2 g}{9v}$

Big drop: $\eta = \frac{2R^2 g}{9V}$

From these two equations:

$$\frac{r^2}{v} = \frac{R^2}{V} \quad \Rightarrow \quad \frac{r^2}{v} = \frac{R^2}{V}$$

$$V = \frac{R^2}{r^2}v = \frac{(1.26)^2\, r^2}{r^2}\ v = 1.5876\ v$$

Q 3: A gas bubble of diameter 2cm, rises steadily through a solution of density $1.75\ g/cm^3$ at the rate of $0.35\ cm/sec$. Calculate the coefficient of viscosity of the solution. (Neglect the density of the gas).

Solution:

$$\eta = \frac{2r^2(\rho - \rho_f)g}{9v}$$

$$\eta = \frac{2 \times (1)^2 \times 1.75 \times 980}{9 \times 0.35} = 1088.88\ poise$$