Semester I (Question bank)

1. List all topologies on the set $X=\{1, 2, 3\}$.
2. List all topologies of cardinality seven on the set $X=\{1, 2, 3, 4\}$.
3. How to define the usual topology on $\Re^2$.
4. Is there any way to define the right ray topology on $\Re$?
5. Let $X\neq\phi$, and a be a fixed element in X, define

$$\tau_a=\{G\subseteq X|a\notin G \vee G=X\}.$$

   Is τ a topology on X?

6. Consider the natural numbers N, define,

   $\tau =\{\{m, m+1, \ldots\}|m\in N\}\cup\{\phi\}$. Is τ a topology on N? Explain your answer.

7. Consider $\Re$, let τ be the set of all closed intervals with ϕ and $\Re$. Is τ a topology on $\Re$? Explain your answer.
8. In a topological space (X,τ), a subset $A\subseteq X$ is called clopen if it is both open and closed in X. Give an example for a topological space for which some of its subsets are clopen.
9. Find $N(3)$, $N(18)$ and $N(100)$ in Exercise 6.

10. Let $X=\{1, 2, 3, 4\}$ and $\tau=\{\phi,\{2,4\}, \{1,4\}, \{4\}, \{1,2,4\}, X\}$. Find $W(\tau)$.
11. Let $X=\{a, b, c, d, e\}$ and $\mathcal{A}=\{\{a, c\}, \{b, c, e\}, \{d, e\}\}$. Find the topology τ on X generated by $\mathcal{A}$.
12. Let $Y=\{1, 2, 3, 4, 5, 6\}$ and $\mathcal{A}=\{\{2, 6\}, \{1, 2, 4\}, \{4, 5\}\}$. Find the topology τ on Y generated by $\mathcal{A}$.
13. Let $X=\{1, 2, 3, 4, 5\}$ and $\mathfrak{B}=\{\{2,3,4\}, \{3,4,5\}\}$. Is there any topology on X, for which $\mathfrak{B}$ is a base?
14. Find a subbase for the right ray topology on $\Re$.
15. In a topological space (X, τ), a subclass $\mathfrak{B}_x \subseteq \tau$ is called a local base for a point $x\in X$, if the following conditions satisfy
    i. $x\in B, \forall B\in\mathfrak{B}_x$.

ii. $G\in\tau \wedge x\in G \rightarrow \exists\ B\in\mathfrak{B}_x$ such that $G\subseteq B$.

I. Let (X, τ) be a topological space and $y\in X$, if $\mathfrak{B}$ is a base for τ, then show that the following class of sets

$\Psi=\{B\in\mathfrak{B}: y\in B\}$ is a local base for $y\in X$. In other words, show that $\Psi=\mathfrak{B}_y$.

II. For Exercise 2, find $\mathfrak{B}_a$, $\mathfrak{B}_c$ and $\mathfrak{B}_e$.

16. Is closure operator bijective?
17. Let (X, τ) be a topological space. Find a result analogous to Theorem 4.14 in term of "*ext*".
18. In Exercise 6, Page 10, find
    a. $drv(\{4, 5, 6, \ldots\})$, $drv(Z_e)$, $drv(\{10, 5, 16\})$
    b. $clo(\{4, 5, 6, \ldots\})$, $clo(Z_e)$, $clo(\{10, 5, 16\})$
    c. $int(\{4, 5, 6, \ldots\})$, $int(Z_e)$, $int(\{10, 5, 16\})$
    d. $bou(\{4, 5, 6, \ldots\})$, $bou(Z_e)$, $bou(\{10, 5, 16\})$
19. Let τ_1 and τ_2 be two topologies on X for which τ_1 is coarser than τ_2. Let $A\subseteq X$. Prove or disprove
    a. Any accumulation point of A in (X, τ_1) is an accumulation point of A in (X, τ_2).
    b. Any accumulation point of A in (X, τ_2) is an accumulation point of A in (X, τ_1).
20. Let (X, τ) be a topological space and $A\subseteq X$. Prove that

$$bou(A)=clo(A)\cap clo(A^c).$$

21. Discuss the dense in Exercise 3.
22. In left-ray topology, let $Y=[0, 1]$. Describe the subspace of Y.
23. Let (Y, τ_Y) be a subspace of a topological space (X, τ) and $A\subseteq Y$. Is there any relation between $int(A)$ and $int_Y(A)$, where $int_Y(A)$ is the interior of A in (Y, τ_Y).
24. Let (Y, τ_Y) be a subspace of a topological space (X, τ) and $(Z, (\tau_Y)_Z)$ be a subspace of (Y, τ_Y). Then $(Z, (\tau_Y)_Z)$ is a subspace of (X, τ).