Semester II (Question bank)

1. A map f: $(X,\tau)\to(Y,\rho)$ is continuous at $x\in X$ if and only if for any open set H in Y containing $f(x)$, there exists an open set G_x in X containing x such that,

$$f(G_x)\subseteq H$$

2. Let f: $(X,\tau)\to(Y,\rho)$ and g: $(Y,\rho)\to(Z,\sigma)$ be continuous maps, then $g\circ f : (X,\tau)\to(Z,\sigma)$ is continuous.
3. Let f: $(X,\tau)\to(Y,\rho)$ be a map and $\mathfrak{B}$ be a base for τ. Then f is open if and only if $f(B)$ is open, for all $B\in\mathfrak{B}$.
4. Is $f : (R,U^1)\to (R,U^1)$ an open map?
5. Under the usual topology on R, prove that any closed interval $[a, b]$ is homeomorphic to $[0,1]$.
6. Neighbourhood of a point is a topological property.

   Is τ a topology on X?
7. Let $X=\{a, b, c, d, e\}$. Find

   7.1 Four different connected spaces of cardinalities 4, 5, 6 and 7.

   7.2 Five different disconnected spaces of cardinalities 3, 4, 5, 6 and 7.
8. Prove or disprove:

   If (X,τ) is a connected space and ρ is coarser than τ, then (X,ρ) is connected.
9. In a topological space (X,τ), the components of X form a partition of X.
10. Prove or disprove:

    If (X,τ) is a compact space and ρ is coarser than τ, then (X,ρ) is compact.
11. The property of T_0 space is hereditary.
12. T_1 space is preserved under a bijective open map.
13. If g,h: $(X,\tau)\to(Y, \rho)$ are continuous mappings for (Y, ρ) is a Hausdorff space, then, the set,

$$F=\{x\in X|\ f(x)=g(x)\} \text{ is closed}$$

14. Prove or disprove:

Every regular T_0 space is a T_3 space.

15. The property of being first countable space is a topological property.

16. Any subspace of a second countable space is a second countable space.