Salahaddin University-Erbil	College of Science	Mathematics Department
Semester I-Year 4	Final Exam (1^{st} Trial)	General Topology
Date: 17/12/2023		Duration time: 120 minutes
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Remark.
During this exam sheet, the pair (X, τ) is a topological space.

Q.1. (30 marks) Select **T** for true and **F** for false statements (**Answer in order**)

1. A topology τ on a set X is discrete iff $\{x\} \in \tau$, for some $x \in X$.
2. In (X,τ), for any $x \in X$, the neighbourhood system $\mathcal{N}(x) \neq \emptyset$.
3. If $X \neq \emptyset$ and $\mathcal{A} \subseteq Power(X)$, then $\mathcal{A}$ generates a unique topology on X for which $\mathcal{A}$ is a subbase.
4. In a trivial topology $(X, \mathcal{I})$, if $\emptyset \neq A \subseteq X$, then $drv(A) \neq \emptyset$.
5. If (X,τ_1) and (Y,τ_2) are two topological spaces, then $(X \cap Y, \tau_1 \cap \tau_2)$ is a topological space.
6. In $(\mathbf{R}, U^1)$, "$clo(I_{rr}) = I_{rr}$", where I_{rr} is the set of irrational numbers.
7. In (X,τ), for any $Y \subseteq X$, we can define the subspace (Y, τ_Y).
8. In (X,τ_{cof}), any subset of X is closed iff it is finite.
9. In (X,τ). For any $A \subseteq X$, "$clo(A)$"and "$int(A)$"are disjoint sets.
10. In $(\mathcal{R},\tau_{right})$, if A is infinite and $a \in A$, then $a \in drv(A)$.
11. In (X,τ), "*bou*" is a 1-1 operator.
12. Two equal topologies on a set X might have distinct bases.
13. Any closed set in (X,τ) is a dense in itself.
14. In (X,τ) and $A, B \subseteq X$, if $A \neq B$, then $int(A) \neq int(B)$.
15. In $(\mathbf{R}, U^1)$, the set $\left\{\frac{1}{n};\ n \in Z^+\right\}$ is not a closed set.

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Q.2. (10 marks)
Consider the topological space (Z^+, τ), where,

$$\tau = \left\{\{m, m+1, \cdots\};\ m \in Z^+\right\} \cup \left\{\emptyset\right\}.$$

1. List the open sets that contain 7.
2. List the closed sets that contain 7.
3. Find $ext(\{4, 5, 7, 13\})$.
4. Find $clo(\{8, 15, 23, 103\})$.
5. Find the subspace τ_Y, where $Y = \{5, 8, 10, 11, 12, 13, \cdots\}$.

⇒*Additional questions are presented on the reverse side.*

Q.3. (10 marks)

(a) Let (X,τ) be a topological space for which δ is a subbase and $Y \subseteq X$. Show that

$$\delta_Y = \{s \cap Y; s \in \delta\},$$

is a subbase for the subspace (Y, τ_Y).

(b) Let (Y,τ_Y) be a subspace of a topological space (X,τ) and $(Z,(\tau_Y)_Z)$ be a subspace of (Y,τ_Y). Prove that $(Z,(\tau_Y)_Z)$ is a subspace of (X,τ).

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Q.4. (10 marks) (***Prove or disprove***)(**Select Only Two**)

(a) The union of two topologies on a set $X \neq \emptyset$, is a topology on X.

(b) In (X,τ). For $A \subseteq X$.

$$A \in \tau \iff bou(A) \cap A = \emptyset$$

(c) If (Y, τ_Y) is a subspace of the topological space (X,τ) and $A \subseteq Y$, then

$$ext_Y(A) = Y \cap ext(A).$$

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GOOD LUCK ON YOUR EXAM