Salahaddin University-Erbil	College of Science	Mathematics Department
Semester II-Year 4	Final Exam (1^{st} Trial)	General Topology II
Date: 5/5/2024		Duration time: 120 minutes
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Remark.
During this exam sheet, the pair (X,τ) and (Y,ρ) are topological spaces.

Q.1. (30 marks) Select **T** for true and **F** for false statements (**Answer in order**)

1. Second Axiom space is hereditary.
2. If a map $f:(X,\tau)\to(Y,\rho)$ is bijective, then f is homeomorphism if and only if $f(int(A))=int(f(A))$, for all $A\subseteq X$.
3. If (X,τ) is compact, $A\subseteq X$ such that $A\subseteq drv(A)$, then A is compact.
4. Any T_2-space is a T_3-space.
5. First countable space is a topological property.
6. The union of two connected set is connected.
7. If (X,τ) is a T_1-space, then $x,y\in X, x\neq y\to clo(x)\neq clo(y)$.
8. Normality is not hereditary.
9. Any two discrete spaces are homeomorphic.
10. The usual topology is a second axiom space.
11. T_0-space is preserved under a bijective continuous map.
12. Any regular space is a Hausdorff space.
13. In a Hausdorff space (X,τ), the set $\{a_1,\cdots,a_n\}$ is closed.
14. Trivial topology is regular and normal.
15. The projection map on (R^2,U^2) is closed but not open.

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Q.2. (10 marks) (**Select Only Two**)

(a) Let (X,τ) be a topological space such that

$$G\in\tau \text{ and } F \text{ is a closed set in } X \text{ for which } F\subseteq G\to\exists H\in\tau \text{ such that } F\subseteq H \text{ and } clo(H)\subseteq G.$$

Then prove that (X,τ) is normal.

(b) In (X,τ), if any class $\{F_\alpha;\alpha\in\Delta$, where Δ is an arbitrary set$\}$ satisfies the *finite intersection property*, has the itself a non-empty intersection, then prove that (X,τ) is compact.

(c) Prove that, a map $f:(X,\tau)\to(Y,\rho)$ is continuous if and only if $f^{-1}(int(B))\subseteq int(f^{-1}(B))$, for all $B\subseteq Y$.

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⇒*Additional questions are presented on the reverse side.*

Q.3. (10 marks)

(a) (6 marks) Let $X = \{a, b, c, d\}$, find

1. Three connected topological spaces on X of cardinalities 4, 5 and 6.
2. Three disconnected topological spaces on X of cardinalities 4, 6 and 8.

(b) (4 marks) Prove that if a map $f : (X, \tau) \to (Y, \rho)$ is closed, then

$$clo(f(A)) \subseteq f(clo(A)), \text{ for all } A \subseteq X$$

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Q.4. (10 marks) (***Prove or disprove***)

(a) Every regular T_0 space is a T_3-space.

(b) If $f : (X, \tau) \to (Y, \rho)$ is an open map, then the restriction map $f\ |_A$ is open, $\forall A \subseteq X$.

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GOOD LUCK ON YOUR EXAM