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Solution for Double Beta Decay and Lipkin-Meshkov-Glick models Hamiltonian based on the Recurrence Formula and Jacobi matrix methods

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Abstract

Solutions for the Lipkin -Meshkov -Glick and Double Beta Decay models Hamiltonian are obtained by using the Recurrence Formula and Jacobi matrix. There isn't any restriction on the parameters specifying the strength of the interactions.

Keywords: Pairing interaction, The Recurrence Formula and Jacobi matrix, Strengths of the interactions, Shape phase transitions

1 Introduction

The pairing interaction is an important part of the fermion Hamiltonian responsible for the Superconducting phase in metals and in nuclear matter or neutron stars.[1] Also pairing is the significant effect in the nucleus with un fool level that has even proton and neutron .The main feature of the pairing interaction is that it correlates pairs of particles in time-reversed states.Several models proposed for pairing problem such as Richardson model, Double Beta Decay and Lipkin-Meshkov-Glick. And there are different methods for solving these Hamiltonian as Beth-Ansatz, QRPA,RPA, Hartree-Fock.[2,3,4] .In this paper we overview Double Beta Decay and Lipkin-Meshkov-Glick models then Based on the recursion relation formulas and the Jacobi matrix from the theory of orthogonal polynomials, a numeric solution for these Hamiltonian models is given, without considering any restriction on the parameters specifying the strengths of the interactions.

2 Double Beta Decay

This decay is one of the weak rare nuclear processes and happens in the more than sixty isotopes in the periodic table. But experimentally happens only between ten of them including ;

T^{128}, Cd^{116},Mo^{100},Zr^{96},Se^{82},Ge^{76},Ca^{48},Ca^{238},Nd^{150},Te^{130}

this decay can happen in three faces. 1-$\beta\beta2\upsilon$: Neutrino less double beta decay is one of the best probes for physics beyond the standard model of electroweak interactions. Its existence is tied to fundamental aspects of particle physics such as lepton number no conservation, neutrino mass, right-handed currents in the electroweak interaction, a massless Goldstone boson Majoron, the structure of the Higgs' sector and super symmetry. 2-$\beta\beta0\upsilon$: Double Beta decay is transition between a nucleus of charge Z and mass number A(with both A and Z even) and one of the same mass and charge Z+2 . 3-$\beta\beta0\upsilon\chi$: A model many-body Hamiltonian describing a heterogonous system of Paired protons and paired neutrons and interacting among themselves through Monopole particle-hole and monopole particle-particle interactions is used to Study the double beta decay of Fermi type.The model Hamiltonian is transformed in a many body quasiparticle operator which

$H = \epsilon(\mathrm{N}_p + N_n) + \lambda_1 a^\dagger a + \lambda_2(a^\dagger a^\dagger + aa)$ (2-1)

Where N_P and N_n are proton and neutron quasi particle number operators,respectively,while A, $A^\dagger$ stand for proton neutron pairing operators built up with the quasi particle operators

$$A^\dagger = [a_p^\dagger a_n^\dagger]_{00}, \qquad A = (A^\dagger)^\dagger \tag{2-2}$$

This Hamiltonian is the Hamiltonian that is set mixture of proton and neutron sphere shell model and obtain from particle-particle and particle-hole proton-neutron pairing interaction. If we delete the sentence with coefficient λ_1 it defining the LMG Hamiltonian [3,4].

3 Lipkin-Meshkov-Glick model, LMG

This model is offered in 1965 for explaining phase transformation in nucleus. Then is used for quantum information and quantum phase transformation. In this model,N particles can distribute themselves on two N-fold degenerate levels distinguished by a quantum number σ with $= \pm 1$ .Let $a_{p\sigma}^\dagger$ be fermion creation -annihilation operator for a particle in the P state and level with P=1,2,. . . .,N. The Hamiltonian of the LMG model can be written as [5,6]

$$H = \varepsilon \sum_{p,p'=1,2,..N;\sigma=\pm 1} \sigma a_{p\sigma}^\dagger a_{p\sigma} + V \sum_{p,p'=1,2,..N;\sigma=\pm 1} a_{p\sigma}^\dagger a_{p'\sigma}^\dagger a_{p'-\sigma} a_{p-\sigma} + W \sum_{p,p'=1,2,..N;\sigma=\pm 1} a_{p\sigma}^\dagger a_{p'-\sigma}^\dagger a_{p'\sigma} a_{p-\sigma} \tag{3-3}$$

Where V and W are parameters specifying the strengths of the interactions.The interaction term V scatters a pair of particles across the Fermi level, it is a two particle hole interaction.

The term proportional to W scatters one particle up and another down. By introducing the so called pseudo spin operators

$$J_+ = \sum_p a^\dagger_{p+} a_{p-}, \quad J_- = \sum_p a^\dagger_{p-} a_{p+}, \quad J_z = 1/2 \sum_{p\sigma} \sigma a^\dagger_{p\sigma} a_{p\sigma} \tag{3-4}$$

That follows from the following relations

$$[J_+, J_-] = J_z, \quad [J_z, J_\pm] = \pm J_\pm \tag{3-5}$$

In the SU(2)representation Hamiltonian be written as

$$H = \epsilon J_z + \frac{1}{2} V (J_+^2 + J_-^2) + \frac{1}{2} W (J_+ J_- + J_- J_+) \tag{3-6}$$

4 Spectrum distribution, Recurrence Formula and Jacobin matrix Formula

For studying shape phase transition and finding critical points that transition happen there , we suppose unit vector $|\phi_i\rangle$ with the following representations .

$$|\phi_i\rangle = \begin{cases} |J, J-2k_i\rangle & \text{for } k_i = 0, 1, ..., J \\ |J, J-(2k_i+1)\rangle & \text{for } k_i = 0, 1, ..., J-1 \end{cases}. \tag{4-7}$$

We act the Hamiltonian H on these unit vectors and find the coefficients according to this relation [7]

$$H|\phi_i\rangle = \beta_{i+1}|\phi_{i+1}\rangle + \alpha_i|\phi_i\rangle + \beta_i|\phi_{i-1}\rangle \qquad i = 0, 1, ..., n. \tag{4-8}$$

For the first representation in the LMG model we have:

$$\begin{aligned} H|J, J-2k_i\rangle &= (2\epsilon(J-2k_i) + 2WJ + 8Wk(J-k_i))|J, J-2k_i\rangle + \\ &\hbar^2 V \sqrt{4k_i(2k_i-1)(2(J-k_i)+1)(J-k_i+1)}|J, J-2k_i+2\rangle + \\ &\hbar^2 V \sqrt{4(J-k_i)(2J-2k_i-1)(k_i+1)(2k_i+1)}|J, J-2k_i-2\rangle. \end{aligned} \tag{4-9}$$

The appropriate coefficients will be

$$\alpha_{k_i} = 2\epsilon(J-2k_i)+2wJ+8Wk_i(J-k_i), \qquad \beta_{k_i} = \hbar^2 V\sqrt{4k(2k_i-1)(2(J-k_i)+1)(J-k_i+1)}. \tag{4-10}$$

and for the second unit vectors $|J, J-(2k_i+1)\rangle$ we have

$$\begin{aligned} H|J, J-(2k_i+1)\rangle &= (2\epsilon(J-2k_i-1)+2W(J+(2k_i+1)(2J-2k_i-1))|J, J-(2k_i+1)\rangle+ \\ &\hbar^2 V\sqrt{4k(J-k_i)(2J-2k_i+1)(2k_i+1)}|J, J-(2k_i+1)+2\rangle+ \\ &\hbar^2 V\sqrt{(2J-2k_i-1)(2k_i+2)(2k_i+3)}|J, J-(2k_i+1)-2\rangle. \end{aligned} \tag{4-11}$$

Therefore, the coefficients α_{k_i} and β_{k_i} are given by

$$\alpha_{k_i} = (J-2k_i-1)+2W(J+(2k_i+1)(2J-2k_i-1), \qquad \beta_{k_i} = \hbar^2 V\sqrt{4k(J-k_i)(2J-2k_i+1)(2k_i+1)}. \tag{4-12}$$

Using $2\beta\beta\upsilon$ model for the nucleus with Z=4, j=9/4,$\epsilon = 1mev, N = 6$.coefficients λ_1 and λ_2 will be

$$\lambda_1 = o.52 - 0.96k \qquad \lambda_2 = 0.48(0.5+k)(4\text{-}13)$$

for the even states

$$\alpha_{k_i} = 4k_i +_1 (2k_i - (k_i(2k_i-1))/5), \qquad \beta_{k_i} = \lambda_2(2k_i(2k_i-1)(1-(2k_i-1)/10)(1-(k_i-1)/5)) \tag{4-14}$$

And for the odd states

$$\alpha_{k_i} = 4k_i +_1 (2k_i - (k_i(2k_i+1))/5)\ , \qquad \beta_{k_i} = \lambda_2(2k_i(2k_i+1)(1-(2k_i-1)/10)(1-k_i/5)(4\text{-}15)$$

As we have in ref.[7]

$$x\begin{pmatrix} Q_0(x) \\ Q_1(x) \\ Q_2(x) \\ . \\ . \\ . \\ . \\ Q_{n-1}(x) \end{pmatrix} = \begin{pmatrix} \alpha_0 & \beta_0 & 0 & 0 & . & . & . & 0 \\ \beta_0 & \alpha_1 & \beta_1 & 0 & . & . & . & 0 \\ 0 & \beta_1 & \alpha_2 & \beta_2 & . & . & . & 0 \\ . & . & . & . & & & & 0 \\ . & . & . & . & & & & . \\ . & . & . & . & & & & . \\ . & . & . & . & & & & . \\ 0 & 0 & 0 & 0 & . & . & . & \alpha_{N-1} \end{pmatrix} \begin{pmatrix} Q_0(x) \\ Q_1(x) \\ Q_2(x) \\ . \\ . \\ . \\ . \\ Q_{N-1}(x) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ \beta_{N-1}Q_N(x) \end{pmatrix} \tag{4-16}$$

If x be a zero of $Q_N(x)$, say $x = x_i$ then we obtain $x_iQ(x_i) = JQ(x_i)$ where J is called Jacobi matrix[7] and the eigenvalues $x_1, ..., x_N$ of that are the zeros of $Q_N(x)$ and the eigenvector corresponding to x_i is $(Q_0(x_i), Q_1(x_i), ..., Q_{N-1}(x_i))^\dagger$. If we choose $x = x_i$ then the end matrix will be zero, therefore the coefficients α and β are equal to the coefficients in the recursion relations.

$$Q_0(x) = 1, \qquad Q_1(x) = x - \alpha_1,$$

$$xQ_k(x) = \beta_{k+1}Q_{k+1}(x) + \alpha_{k+1}Q_k(x) + \beta_kQ_{k-1}(x) \quad k \geq 1. \tag{4-17}$$

5 Conclusions and Discussions

In this article we got a solution for the double beta decay and LMG models. Graphs show the pairing transition critical points from normal nucleus to deformed nucleus. There is quantum phase transition in these points.Quantum phase transition shows that the ground state becomes degenerate and a macroscopic change in the ground state energy take place.in this points the ground state and the first excited state come near together and interchange then level crossing take place. There are two types of phase transition in nuclei: the phase transition to super fluidity and to deformation. Shape transition that is shown here has important role in determining their properties such as quadrupole moment, nuclear size and isotope shift. This take place as some control parameters vary along an isotopic (isotonic) chain, for example, brings the system from a spherical to a deformed region. Level-crossing take place in different levels and in the LMG model it has seen more in higher level in the stronger interaction regime. But in in the double Beta decay level-crossing has been seen in the all levels. Comparing these method with other methods for example Beth-Ansatz ,RPA,QRPA and Hartree-Fock [3,4,5] that have many complicated math formulas we can deduce that spectrum distribution ,recursion relations and Jacobi matrix have more precision and can do desire accounts rapidly. Graphs can be drawn and studied for every N easily.The LMG model was conceived as a test model in nuclear physics. It is simple enough to be solved but it is yet nontrivial. For that reason, since it was established has been used to validate many fermion approximation methods like Hartree- Fock [8]. our high levels is upper than the levels obtain from Bethe ansatz and Hartree- fock methods (Fig.5,6)and the low level is lower (Fig.7), this indicates that our method gives better accuracy with respect to the other methods such as Bethe ansatz [3], and infinite- dimensional algebraic [6] that has sophisticated math, Recurrence Formula and the Jacobi matrix is simple.

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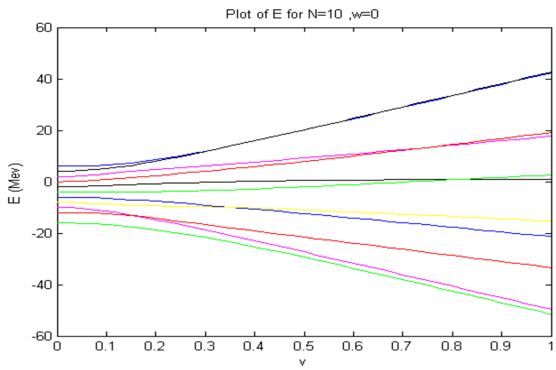

Figure 1: E for N=10 and w=0

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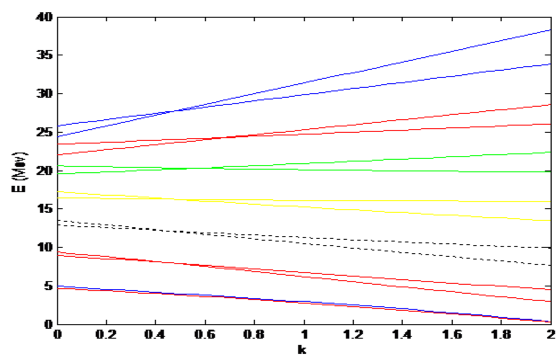

Figure 2: E for Double Beta Decay

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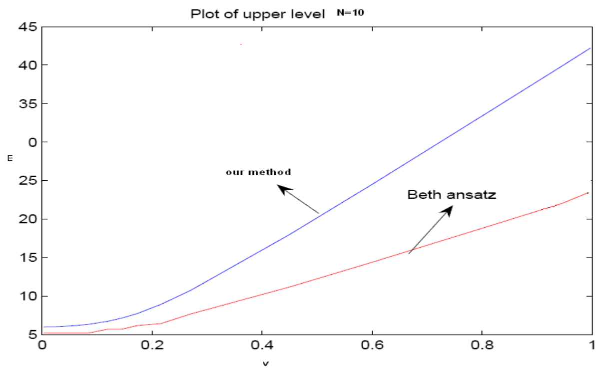

bet ansatz-spec.png

Figure 3: Shows E as a function of the parameter V for 10 particles, W=0,upper level for both Beth ansatz and spectral

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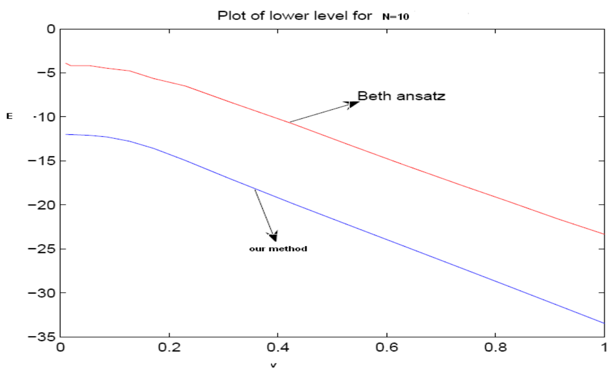

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Figure 4: Shows E as a function of the parameter V for 10 particles, W=0, Lower level for both Beth ansatz and spectral

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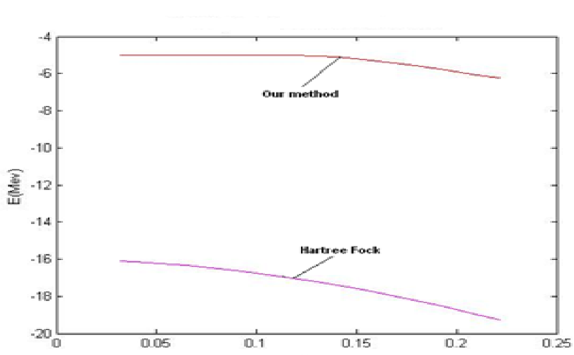

Hartree-spec.png

Figure 5: Shows E as a function of the parameter $\chi = \frac{V}{\varepsilon}(\Omega - 1)$ for 10 particles, W=0 Lower level for both Hartree Fock and spectral

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