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 **Expectation value for double beta decay Hamiltonian model**

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  **Introduction**

In nuclear physics , double beta decay is a type of radioactive decay in which two neutrons are simultaneously transformed into two protons , or vice versa , inside an atomic nucleus. As in single beta decay, this process allows the atomr to move closer to the optimal ratio of protons and neutrons. Because of this transformation, the nucleus emits two detectable beta particle, which are electrons or positrons.

The literature distinguishes between two types of double beta decay: ordinary double beta decay and neutrinoless double beta decay. In ordinary double beta decay, which has been observed in several isotopes, two electrons and two electron antineutrinos are emitted from the decaying nucleus. In neutrino less double beta decay, a hypothesized process that has never been observed, only electrons would be emitted.

The expectation value of an observable is an important concept in quantum mechanics. In quantum mechanics, the expectation value is the probabilistic expected value of the result (measurement) of an experiment. It can be thought of as an average of all the possible out comes of a measurement. It is a fundamental concept in all areas of quantum physics. According to Born the wave function has probabilistic interpretation, therefore it is essential to calculate the average or expectation value of any dynamical quantity defined by the wave function. In physics such dynamical quantities are space-coordinates, momentum and energy of the system. The average or expectation value of a dynamical quantity is the mathematical expectation for the result of a single measurement. It may be defined as the average of the result of a large number of measurements on independent systems. The study of two- level systems has been a topic of interest since the rst steps in the development of quantum mechanics. The main advantage of these models is that they can be numerically diagonalized for very large dimensions and, at the same time, they can model realistic quantum many-body systems. Typical examples are the JaynesCummings model of quantum optics [1], the Vibron Model (VM) of quantum chemistry 2), the two-level pairing model in condensed matter [3] and in muelear physics [4), the Lipkin-Meshikov-Glick model (LMG) 5, 6, 7) and the Interacting Boson Model (IBM) (8) of nuclear structure. While some of these models describe two-level fermion systems, the model Hamiltonian can always be written in terms of SU(2) pseudo-spin operators. Subsequently, the spin Hamiltonian can be expressed in terms of bosons using either the nite Schwinger representation or the innite Holstein-Primako representation of the SU(2) algebra. An example is the LMG model which has recently been newly revived as a model of quantum spins with long-range interactions to investigate the relationship between entanglement and quantum phase transitions (QPTS) [9, 10, 11, 12, 13, 14, 15). In its boson representation, it has also been recently used as a simplied model to describe the Josephson

eect between two Bose-Einstein condensates 16. Physicists frequently use the harmonic oscillator for initial concepts. The model plays a vital role in physics. It is useful for describinge the dynamics of many physical systems. It 11 can model the bond in a molecule as a spring connecting two atoms and use the harmonic oscillator expression to describe the potential energy for the periodic vibration of the atoms. The isotropic three- dimensional harmonic oscillator allows analytical solutions. Many nuclear problems are often approximated by using filed or liquid drop models to study the shape and spectrum of low-lying single particlesle and collective excitations [1-3]. A goal of nuclear- physics is to explain the properties of nuclei using mathematical models of their structure and internal motion. Sometimes, they combine oscillators for expanded purposes. One of these is the pairing interaction. Nuclear pairing plays an important role in differnt single- particle and collective aspects of nuclear structures. The pairing interaction is an integral part of the fermion Hamiltonian responsible for the superconducting phase in metals, nuclear matter or neutron stars [4]. Ultrasmall superconducting grains and atomic nuclei pair in their corresponding finite systems [5]. In addition, pairing is the significant effect in the nucleus 19 with un-fool level that has even proton and neutron. The primary feature of the pairing interaction is the correlation of pairs of particles in time- reversed states. Several models are proposed for pairing, such as Richardson model, double beta decay and LMG. The LMG model, in its simplest version, describes two shells for nucleons and interaction between them in different shells. There is a traditional testing ground for new approximation techniques, that are numerically solvable [6, 7]. Further, there are a variety of standard met hods for calculating nuclear structure that solve these models Hamiltonian such as Bethe-Ansatz, QRPA, RPA and 17 Hartree-Fock [7, 8, 9]. The structure of the article is the following. In section II, we review the harmonic oscillator. Section III, discusses the LMG model, and section IV presents the double 13 beta decay models, Based on the recursion relation formulas and the Jacobi matrix from the theory of orthogonal polynomials, a numeric solution for these Hamiltonian models presented

**Hamiltonian**

Hamiltonian function, also called Hamiltonian, mathematical definition introduced in 1835 by Sir William Rowan Hamilton to express the rate of change in time of the condition of a dynamic physical system—one regarded as a set of moving particles. The Hamiltonian of a system specifies its total energy—i.e., the sum of its kinetic energy (that of motion) and its potential energy (that of position)—in terms of the Lagrangian function derived in earlier studies of dynamics and of the position and momentum of each of the particles.

The Hamiltonian function originated as a generalized statement of the tendency of physical systems to undergo changes only by those processes that either minimize or maximize the abstract quantity called action. This principle is traceable to Euclid and the Aristotelian philosophers.

Figure (1.1) Schematic illustration of the different types of nucleon pairs with orbital angular momentum *L*=0. The valence neutrons (blue) or protons (red) that form the pair occupy timereversed orbits (circling the nucleus in opposite direction). . If the nucleons are identical they must have anti-parallel spins—a configuration which is also allowed for a neutron–proton pair (top) the configuration with parallel spins is only allowed for a neutron–proton pair

(bottom). [18] we can wright the Hamiltonian of springs by quantum operators X and P. then we can write them with simpler operators a and aT

 **Hamilton for spring**

**H=k+v**

**K:is a kainatik energy figure(1.1)**

V: is potional energy

-

mgx







H=Ʀ w (a

T

a+







**a|n>=**$\sqrt{n}$**|n-1>**

**a+ |n=**$\sqrt{n+1}$ **|n+1>**

**H=ħw(a+ a+**$\frac{1}{2}$**)=**

**a+a|n = a+ \*** $\sqrt{n}$ **|n-1 >**

**=**$\sqrt{n }$ **a+ |n-1>=**$\sqrt{n }$ **\*** $\sqrt{n-1+1 }$**|n-1+1>**

**=** $\sqrt{n }$ **\***$\sqrt{n }$**|n>**

**=n|n> ……………………………. 1**

**mg**$\sqrt{\frac{ħ}{2mw }} $**(a+a+ )|n>**

**mg** $\sqrt{\frac{ħ}{2mw }}$**(**$\sqrt{n}$ **|n-1 >+**$\sqrt{n+1}$**|n+1>)**

**mg**$\sqrt{\frac{ħ}{2mw }}$**\*** $\sqrt{n} $**|n-1>+mg**$\sqrt{\frac{ħ}{2mw }}$ **\***$\sqrt{n+1}$**|n+1 >**

**<n|H|n>=?**

**<n|ħw(a+a+**$\frac{1}{2}$**)|n>=ħw<n|a+ a|n>+ħw<n|**$\frac{1}{2}$**|n>**

**ħw<n|n|n>**

**ħw<n|n>**

**<n|H|n> = n ħw+**$\frac{ħw}{2}$ **= ħw(n+**$\frac{1}{2}$**)**

**PAIRING**

lies at the heart of nuclear physics and the quantum many-body problem in general. In infinitely

extended nuclear systems, such as neutron star matter and nuclear matter, the study of superfluidity and pairing has a long history.The pairing interaction is the part of the fermion Hamiltonian responsible for the superconducting phase in metals and in nuclear matter or neutron stars.It correlates pairs of particles in time-reversed states.

The pairing concept In 1911 Kamerlingh Onnes discovered superconductivity in condensed matter systems, and its microscopic explanation came about through the highly successful pairing theory proposed in 1957 by Bardeen, Cooper, and Schrieffer (BCS) (Cooper et al., 1957). A series of first applications to nuclear structure followed (Belyaev, 1959; Bohr et al.,1958; Migdal, 1959). Then It can be traced back to the seniority scheme introduced by Racah in atomic physics and its physical significance was first realized in the study of

superconductivity in macroscopic systems by Bardeen, Cooper and Schrieffer. In 1958, Bohr Mottelson and Pines observed that the gap in the excitation spectrum of even–even nuclei might be due to correlations between the nucleons in the nucleus that are similar to those between electrons in a superconductor.

Over fifty years ago, Mayer (Mayer, 1950) pointed out that a short-ranged, attractive, nucleonnucleon interaction would yield J = 0 ground states. The realistic bare nucleon-nucleon potential indeed contains short-range attractive parts (particularly in the singlet-S and triplet-P channels) that give rise to pairing in infinite nuclear matter and nuclei.

The BCS theory also generalized the seniority coupling scheme in which pair-wise coupling of equivalent nucleons to a state of zero angular momentum takes place. The scheme had been developed during years previous to the discovery of BCS theory (Mayer, 1950; Racah, 1942; Racah and Talmi, 1953)

 Figure{1.2}



 **Gamma decay**

Gamma ray is electromagnetic radiations (photons) of nuclear origin and short wave length typically in the range of 104 fm and 100 fm corresponding to energies 0.1 to 10 mev .

When a nucleus disintegrates by emitting alpha, beta or any other kind of particle, it is usually left in an excited state. The excited nucleus does not have enough energy to emit another particle, or if the decay by emission of another particle is slow, the nucleus decays by electromagnetic radiation.

When a nucleus makes transition from an initial excited state (Ei) to the final state (Ef) , the energy difference ∆E between the two states is equal to energy of the gamma ray Eγ ;

  ***∆E=Ei-Ef***

 ***E*γ *=Ei-Ef***

 **Alpha decay**

If apparent nuclear **A Z X**disintegrates into a daughter nucleus **A-4Z-2 X** plus alpha particle**42He**The process is called alpha decay

**ZA X**  **A-4Z-2 Y + 42He**

**23892 U**  **23490T*h* + 42He**

Alpha particle or (helium nucleus **42He** )composed of two proton and two neutron

Most alpha emitters are heavy nucleus with mass number A > 150

An alpha particle is double magic number nucleus Z=2 , N=2 all nucleons in S1/2 She’ll and it has extraordinary stability

The total spin of alpha particle is zero and have even parity

Double Beta Decay

This decay is one of the weak, rare nuclear processes that occur in the more than sixty isotopes in the periodic table. Experiment ally, however, it occures in only ten of the isotopes; including:

 **T128, Cd116 ,Mol00 ,Zr96 ,Se82,Ge76 ,Ca48 ,Ca238 ,Nd150 ,Tel30**

This decay can occur in three faces. 1) ββ2v: the neutrino less double beta decay is one of the best probes for physics beyond the standard model of electroweak interactions. Its existence is tied to the fundamental aspects of particle physics such as the lepton number, no conservation, neutrino mass, right-handed currents in the electroweak interaction, a massless Goldstone boson Majoron, the structure of the Higgins sector and super symmetry. 2) ββ0v: the double beta decay is a transition between a nucleus of charge Z and mass number A( with both A and Z even) and one of the same mass and charge Z+2. 3) ββ0vx: A many-body model Hamiltonian describing a heterogeneous system of paired protons and paired neutrons that interact among themselves through monopole particle-hole and monopole particle-particle interactions; this is used to study the double beta decay of Fermi type. The model Hamiltonian is transformed in a many-body quasi-particle operator in which:

H =c(Np + Nn)+γ1a†a+γ2(a†a† + aa)

 Where Np and Nn are proton and neutron quasi-particle number operators,respectively,while A, A† are proton neutron pairing operators built up with the quasi particle operators

 **A† =[a†pa†n ]00 , A= (A†)†**

 **T**his Hamiltonian is a mixture of proton and neutron sphere shell models and resultsf rom a particle-particle and particle-hole proton-neutron pairing interaction. If we delete the sentence with the coefficient A it definines the LMG Hamiltonian.

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