1) A classical harmonic oscillator of mass m and spring constant k is known to have a total energy of E, but its starting time is completely unknown. Find the probability density function, p(x), where p(z)ds is the probability that the mass would be found in the interval dx at x.

2) Suppose there are two kinds of E. coli (bacteria), "red" ones and "green" ones. Each reproduces faithfully (no sex) by splitting into half, red-+red+red or green+green+green, with a reproduction time of 1 hour. Other than the markers "red" and "green", there are no differences between them. A colony of 5,000 "red" and 5,000 "green" E. coli is allowed to eat and reproduce. In order to keep the colony size down, a predator is introduced which keeps the colony size at 10,000 by eating (at random) bacteria.

(a) After a very long time, what is the probability distribution of the number of red bacteria? 161

162 Problems €4 SollLtioru on Thermodynamics €4 Statistical Mechanica

(b) About how long must one wait for this answer to be true?

(c) What would be the effect of a 1% preference of the predator for eating red bacteria on (a) and (b)?

3) (a) Consider a large number of N localized particles in an external magnetic field H. Each particle has spin 1/2. Find the number of states accessible to the system as a function of M,, the z-component of the total spin of the system. Determine the value of M, for which the number of states is maximum. (b) Define the absolute zero of the thermodynamic temperature. Explain the meaning of negative absolute temperature, and give a concrete example to show how the negative absolute temperature can be reached.

4) (a) Give the definition of entropy in statistical physics. (b) Give a general argument to explain why and under what circumstances the entropy of an isolated system A will remain constant, or increase. For convenience you may assume that A can be divided into subsystems B and C which

are in weak contact with each other, but which themselves remain in internal thermodynamic equilibrium.

5) ical meaning briefly but clearly. A two-level system of N = nl +n2 particles is distributed among two eigenstates 1 and 2 with eigenenergies El and E2 respectively. The system is in contact with a heat reservoir at temperature T. If a single quantum emission into the reservoir occurs, population changes n2 --+ n2 - 1 and n1 -+ nl+ 1 take place in the system. For nl >> 1 and n2 >> 1, obtain the expression for the entropy change of (a) the two level system, and of (b) the reservoir, and finally (c) from (a) and (b) derive the Boltzmann relation for the ratio nl/nz.

6) Consider a system composed of a very large number N of distinguishable atoms, non-moving and mutually non-interacting, each of which has only two (non-degenerate) energy levels: 0,s > 0. Let E/N be the mean energy per atom in the limit N -+ 00. (a) What is the maximum possible value of E/N if the system is not necessarily in thermodynamic equilibrium? What is the maximum attainable value of E/N if the system is in equilibrium (at positive temperature, of course)? (b) For thermodynamic equilibrium, compute the entropy per atom, (Prince ton) SIN, as a function of E/N.

7) Consider a system of N non-interacting particles, each fixed in position and carrying a magnetic moment p, which is immersed in a magnetic field H. Each particle may then exist in one of the two energy states E = 0 or E = 2pH. Treat the particles as distinguishable. (a) The entropy, S, of the system can be written in the form S = klnR(E), where k is the Boltzmann constant and E is the total system energy. Explain the meaning of R(E). (b) Write a formula for S(n), where n is the number of particles in the upper state. Crudely sketch S(n). (c) Derive Stirling's approximation for large n: Inn! = nlnn - n by approximating In n! by an integral. (d) Rewrite the result of (b) using the result of (c). Find the value of (e) Treating E as continuous, show that this system can have negative (f) Why is negative temperature possible here but not for a gas in a (CUSPEA) (a)

R(E) is the number of all the possible microscopic states of the n for which S(n) is maximum. absolute temperature. box?

8) onsider a one-dimensional chain consisting of n >> 1 segments as illustrated in the figure. Let the length of each segment be a when the long dimension of the segment is parallel to the chain and zero when the segment is vertical (i.e., long dimension normal to the chain direction). Each segment has just two states, a horizontal orientation and a vertical orientation, and each of these states is not degenerate. The distance between the chain ends is nx. (a) Find the entropy of the chain as a function of x. (b) Obtain a relation between the temperature T of the chain and the tension F which is necessary to maintain the distance nz, assuming the joints turn freely. (c) Under which conditions does your answer lead to Hook's law?

9) Consider an idealization of a crystal which has N lattice points and the same number of interstitial positions (places between the lattice points where atoms can reside). Let E be the energy necessary to remove an atom 174 Problems €4 Sdutio~ on Thermodynam'ca €4 Statistical Mechanics from a lattice site to an interstitial position and let n be the number of atoms occupying interstitial sites in equilibrium. (a) What is the internal energy of the system? (b) What is the entropy S? Give an asymptotic formula valid when (c) In equilibrium at temperature T, how many such defects are there (Princeton) n>> I? in the solid, i.e., what is n? (Assume n >> 1.)

10) (a) Explain Boltzmann statistics, Fermi statistics and Bose statistics, especially about their differences. How are they related to the indistinguishability of identical particles? Statistical Phymca 175 (b) Give as physical a discussi6n as you can, on why the distinction between the above three types of statistics becomes unimportant in the limit of high temperature (how high is high?). Do not merely quote formulas. (c) In what temperature range will quantum statistics

have to be applied to a collection of neutrons spread out in a two-dimensional plane with the number of neutrons per unit area being - 1012/cm2?

11) (a) State the basic differences in the fundamental assumptions under- (b) Make a rough plot of the energy distribution function at two different temperatures for a system of free particles governed by MB statistics and one governed by FD statistics. Indicate which curve corresponds to the higher temperature. (c) Explain briefly the discrepancy between experimental values of the specific heat of a metal and the prediction of MB statistics. How did FD statistics overcome the difficulty?

12) State which statistics (classical Maxwell-Boltzmann; Fermi-Dirac; or Bose-Einstein) would be appropriate in these problems and explain why (semi-quantit atively) : (a) Density of He4 gas at room temperature and pressure. (b) Density of electrons in copper at room temperature. (c) Density of electrons and holes in semiconducting Ge at room temperature (Ge band-gap w 1 volt).

13) A long, thin (i.e., needle-shaped) dust grain floats in a box filled with gas at a constant temperature T. On average, is the angular momentum vector nearly parallel to or perpendicular to the long axis of the grain? Explain.

14) A cubically shaped vessel 20 cm on a side constains diatomic H2 gas at a temperature of 300 K. Each H2 molecule consists of two hydrogen atoms with mass of 1.66~10-’~ g each, separated by - lop8 cm. Assume that the gas behaves like an ideal gas. Ignore the vibrational degree of freedom. (a) What is the average velocity of the molecules? (b) What is the average velocity of rotation of the molecules around an axis which is the perpendicular bisector of the line joining the two atoms (consider each atom as a point mass)? (c) Derive the values expected for the molar heat capacities C, and C, for such a gas.

15) Find and make careful sketch of the probability density, p(E), for the energy E of a single atom in a classical non-interacting monatomic gas in thermal equilibrium.

16) pression E(z) = az2 where z is a coordinate or momentum and can take on all values from -00 to $00. (a) Show that the average energy per particle for a system of such particles subject to Boltzmann statistics will be E = kT/2. (b) State the principle of equipartition of energy and discuss briefly its relation to the above calculation.

17) A system of two energy levels Eo and El is populated by N particles at temperature T. The particles populate the energy levels according to the classical distribution law. (a) Derive an expression for the average energy per particle. (b) Compute the average energy per particle vs the temperature as (c) Derive an expression for the specific heat of the system of N par- (d) Compute the specific heat in the limits T + 0 and T -+ 00.

18) The three lowest energy levels of a certain molecule are El = 0, Ez = E, E3 = 10~. Show that at sufficiently low temperatures (how low?) only levels El, E2 are populated. Find the average energy E of the molecule at temperature T. Find the contributions of these levels to the specific heat per mole, C,, and sketch C, as a function of T.

19) Given a system of two distinct lattice sites, each occupied by an atom whose spin (3 = 1) is so oriented that its energy takes one of three values E = 1,0, -1 with equal probability. The atoms do not interact with each other. Calculate the ensemble average values and ? for the energy U of the system, assumed to be that of the spins only.

20) A piece of metal can be considered as a reservoir of electrons; the work function (energy to remove an electron from the metal) is 4 eV only the 1s orbital (which can be occupied by zero, one, or two electrons) and knowing that the hydrogen atom has an ionization energy of 13.6 eV and an electron affinity of 0.6 eV, determine for atomic hydrogen in chemical equilibrium at T =

300 K in the vicinity of a metal the probabilities of finding H+ , Ho and H- . Give only one significant figure. What value of the work function would give equal probabilities to Ho and H-?

21) Derive an expression for the vibrational specific heat of a diatomic gas as a function of temperature. (Let hwo/k = 0). For full credit start with an expression for the vibrational partition function, evaluate it, and use the result to calculate Cvib. Describe the high and low T limits of Cvib.

22) Consider a heteronuclear diatomic molecule with moment of inertia I. In this problem, only the rotational motion of the molecule should be considered. (a) Using classical statistical mechanics, calculate the specific heat C(T) of this system at temperature T. (b) In quantum mechanics, this system has energy levels h2 . . E. 3 (3 + 1) , j = 0,1,2, ... - - - 21 Each j level is (2j + 1)-fold degenerate. Using quantum statistics, derive expressions for the partition function z and the average energy (E) of this 202 PmMems d Sdutiom on Thermodynamics d Statistical Mechunica system, as a function of temperature. Do not attempt to evaluate these expressions. (c) By simplifying your expressions in (b), derive an expression for the specific heat C(T) that is valid at very low temperatures. In what range of temperatures is your expression valid? (d) By simplifying your answer to (b), derive a high temperature approximation to the specific heat C(T). What is the range of validity of your approximat ion?

23) At the temperature of liquid hydrogen, 20.4K, one would expect molecular HZ to be mostly (nearly 100%) in a rotational state with zero angular momentum. In fact, if H2 is cooled to this temperature, it is found that more than half is in a rotational state with angular momentum h. A catalyst must be used at 20.4K to convert it to a state with zero rotational angular momentum. Explain these facts.

24) A gas of molecular hydrogen H2, is originally in equilibrium at a temperature of 1,000 K. It is cooled to 20K so quickly that the nuclear spin states of the molecules do not change, although

the translational and rotational degrees of freedom do readjust through collisions. What is the approximate internal energy per molecule in terms of temperature units K? Note that the rotational part of the energy for a diatomic molecule is A1(1+ 1) where 1 is the rotational quantum number and A - 90K for H2. Vibrational motion can be neglected.

25) In hydrogen gas at low temperatures, the molecules can exist in two states: proton spins parallel (orthohydrogen) or anti-parallel (parahydrogen). The transition betwen these two molecular forms is slow. Experiments performed over a time scale of less than a few hours can be considered as if we are dealing with two separate gases, in proportions given by their statistical distributions at the last temperature at which the gas was allowed to come to equilibrium. (a) Knowing that the separation between protons in a hydrogen molecule is 7.4~ lo-' cm, estimate the energy difference between the ground state and the first excited rotational state of parahydrogen. Use degrees Kelvin as your unit of energy. Call this energy k60, so that rrors in (a) do not propagate into the other parts of the question. (b) Express the energy difference between the ground and first excited rotational states of orthohydrogen, kO1, in terms of k00. In an experiment to measure specific heats, the gas is allowed to come to equilibrium at elevated temperature, then cooled quickly to the temperature at which specific heat is measured. What will the constant-volume molar specific heat be at: (c) temperatures well above 00 and 01, but not high enough to excite (d) temperatures much below 00 and 01 [include the leading temperature-dependent term]? (e) T = 60/2?

26) Molecular hydrogen is usually found in two forms, orthohydrogen ("parallel" nuclear spins) and parahydrogen ("anti-parallel" nuclear spins). (a) After coming to equilibrium at "high" temperatures, what fraction of H2 gas is parahydrogen (assuming that each variety of hydrogen is mostly in its lowest energy state)? (b) At low temperatures orthohydrogen converts mostly to

parahydrogen. Explain why the energy released by each converting molecule is much larger than the energy change due to the nuclear spin flip.

27) A ”14 nucleus has nuclear spin I = 1. Assume that the diatomic molecule N2 can rotate but does not vibrate at ordinary temperatures and ignore electronic motion. Find the relative abundances of the ortho- and para-molecules in a sample of nitrogen gas. (Ortho = symmetric spin state; Para = antisymmetric spin state). What happens to the relative abundance as the temperature is lowered towards absolute zero? (Justify your answers!)

28) (a) Write down a simple expression for the internal part of the partition function for a single isolated hydrogen atom in very weak contact with a reservoir at temperature T. Does your expression diverge for T = 0, for T # O? (b) Does all or part of this divergence arise from your choice of the zero of energy? (c) Show explicitly any effects of this divergence on calculations of the (d) Is the divergence affected if the single atom is assumed to be confined to a box of finite volume L3 in order to do a quantum calculation of the full partition function? Explain your answer.

29) The average kinetic energy of the hydrogen atoms in a certain stellar (a) What is the temperature of the atmosphere in Kelvins? (b) What is the ratio of the number of atoms in the second excited (c) Discuss qualitatively the number of ionized atoms. Is it likely to atmosphere (assumed to be in thermal equilibrium) is 1.0 eV. state (n = 3) to the number in the ground state? be much greater than or much less than the number in n = 3? Why?

30) a) State the Maxwell-Boltzmann energy distribution law. (b) Assume the earth's atmosphere is pure nitrogen in thermodynamic equilibrium at a temperature of 300 K. Calculate the height above sea-level at which the density of the atmosphere is one half its sea-level value.

31) A circular cylinder of height L, cross-sectional area A, is filled with a gas of classical point particles whose mutual interactions can be ignored. The particles, all of mass rn, are acted on by

gravity (let g denote the gravitational acceleration, assumed constant). The system is maintained in thermal equilibrium at temperature T. Let c, be the constant volume specific heat (per particle). Compute c, as a function of T, the other parameters given, and universal parameters. Also, note especially the result for the limiting cases, T -+ 0, T -+ 00.

32) Ideal monatomic gas is enclosed in cylinder of radius a and length L. The cylinder rotates with angular velocity w about its symmetry axis and the ideal gas is in equilibrium at temperature T in the coordinate system rotating with the cylinder. Assume that the gas atoms have mass rn, have no internal degrees of freedom, and obey classical statistics. (a) What is the Hamiltonian in the rotating coordinates system? (b) What is the partition function for the system? (c) What is the average particle number density as a function of r?

33) Find the particle density as a function of radial position for a gas of N molecules, each of mass M, contained in a centrifuge of radius R and length L rotating with angular velocity w about its axis. Neglect the effect of gravity and assume that the centrifuge has been rotating long enough for the gas particles to reach equilibrium.

34) A paramagnetic system consists of N magnetic dipoles. Each dipole carries a magnetic moment p which can be treated classically. If the system at a finite temperature T is in a uniform magnetic field H, find (a) the induced magnetization in the system, and (b) the heat capacity at constant H.

35) Consider a gas of spin 1/2 atoms with density n atoms per unit volume. Each atom has intrinsic magnetic moment p and the interaction between atoms is negligible. Assume that the system obeys classical statistics. (a) What is the probability of finding an atom with p parallel to the applied magnetic field H at absolute temperature T? With p anti-parallel to H? (b) Find the mean magnetization of the gas in both the high and low (c) Determine the magnetic susceptibility x in terms of p.

36) Consider an ideal quantum gas of Fermi particles at a temperature T. (a) Write the probability p(n) that there are n particles in a given (b) Find the root-mean-square fluctuation ((n - (n))z))'/z in the occupation number of a single particle state as a function of the mean occupation number (n). Sketch the result.

37) Calculate the average energy per particle, E, for a Fermi gas at T = 0, (UC, Berkeley) given that EF is the Fermi energy.

38) Consider a Fermi gas at low temperatures kT << p(O), where p(0) is the chemical potential at T = 0. Give qualitative arguments for the leading value of the exponent of the temperature-dependent term in each of the following quantities: (a) energy; (b) heat capacity; (c) entropy; (d) Helmholtz free energy; (e) chemical potential. The zero of the energy scale is at the lowest orbital.

39) Derive an expression for the chemical potential of a free electron gas with a density of N electrons per unit volume at zero temperature (T = 0 K). Find the chemical potential of the conduction electrons (which can be considered as free electrons) in a metal with N = loz2 electrons/cm3 at T=OK.

40) For Na metal there are approximately 2.6 x conduction electrons/ cm3, which behave approximately as a free electron gas. From these facts, (a) give an approximate value (in eV) of the Fermi energy in Na, (b) give an approximate value for the electronic specific heat of Na at (UC, Berkeley) room temperature.

41) Consider a Fermi gas model of nuclei. Except for the Pauli principle, the nucleons in a heavy nucleus are assumed to move independently in a sphere corresponding to the nuclear volume V. They are considered as a completely degenerate Fermi gas. Let A = N (the number of neutrons) +Z (the number of protons), assume N = 2, and compute the kinetic energy per nucleon, Ekin/A,

with this model. Statistical Phyaca 281 4s 3 The volume of the nucleus is given by V = -RiA, Ro M 1.4 x Please give the result in MeV. cm.

42) Consider a collection of N two-level systems in thermal equilibrium at a temperature T. Each system has only two states: a ground state of energy 0 and an excited state of energy E. Find each of the following quantities and make a sketch of the temperature dependence. (a) The probability that a given system will be found in the excited state. (b) The entropy of the entire collection.

43) N weakly coupled particles obeying Maxwell-Boltzmann statistics may each exist in one oi the 3 non-degenerate energy levels of energies -E, 0, +E. The system is in contact with a thermal reservoir at temperature T. (a) What is the entropy of the system at T = 0 K? (b) What is the maximum possible entropy of the system? (c) What is the minimum possible energy of the system? (d) What is the partition function of the system? (e) What is the most probable energy of the system? (f) If C(T) is the heat capacity of the system, what is the value of Irn C(T)d

44) Find the pressure, entropy, and specific heat at constant volume of an ideal Boltzmann gas of indistinguishable particles in the extreme relativistic limit, in which the energy of a particle is related to its momentum by E = cp. Express your answer as functions of the volume V, temperature T, and number of particle N.

45) Consider a dilute diatomic gas whose molecules consist of non-identical pairs of atoms. The moment of inertia about an axis through the molecular center of mass perpendicular to the line connecting the two atoms is I. Calculate the rotational contributions to the specific heat and to the absolute entropy per mole at temperature T for the following limiting cases: (a) kT >> h2/I, (b) kT << h2/I. Make your calculations sufficiently exact to obtain the lowest order (CUSPEA) non-zero contributions to the specific heat and entropy.

46) Give a qualitative argument based on the kinetic theory of gases to show that the coefficient of viscosity of a classical gas is independent of the pressure at constant temperature.

47) A propagating sound wave causes periodic temperature variations in a gas. Thermal conductivity acts to remove these variations but it is generally claimed that the waves are adiabatic, that is, thermal conductivity is too slow. The coefficient of thermal conductivity for an ideal gas from kinetic theory is k M 1.23CU?il where C, is the heat capacity per unit volume, is is the mean thermal speed, and 1 is the mean free path. What fraction of the temperature variation AT will be conducted away vs X and what is the condition on X for thermal conductivity to be ineffective?

48) A quantity of argon gas (molecular weight 40) is contained in a chamber at To = 300 K. (a) Calculate the most probable molecular velocity. A small hole is drilled in the wall of the chamber and the gas is allowed to effuse into a region of lower pressure. (b) Calculate the most probable velocity of the molecules which escape through the hole. The pressures of the chamber and the region outside the hole are adjusted so as to sustain a hydrodynamic flow of gas through the hole, such that viscous effects, turbulence, and heat exchange with the wall of the hole may be neglected. During this expansion the gas is cooled to a temperature of 30 K. (c) Calculate the velocity of sound c at the lower temperature. (d) Calculate the average flow velocity V at the lower temperature, and compare the distribution of velocities with the original distribution in the chamber.

49) A parallel beam of Be (A = 9) atoms is formed by evaporation from an oven heated to 1000 K through a small hole. (a) If the beam atoms are to traverse a 1 meter path length with less than l/e loss resulting from collisions with background gas atoms at room temperature (300 K), what should be the pressure in the vacuum chamber? Assume a collision cross-section of lo-'' cm2, and ignore collisions between 2 beam atoms. (b) What is the mean time (T) for the beam atoms

to travel one meter? Show how the exact value for 7 is calculated from the appropriate velocity distribution. Do not evaluate integrals. Make a simple argument to get a numerical estimate for 7. (c) If the Be atoms stick to the far wall, estimate the pressure on the wall due to the beam where the beam strikes the wall. Assume the density 394 Problem3 d Solutiotu on Thermodynamics €4 Statistical Mechanics of particles in the beam is lo1'/ cm3. Compare this result with the pressure from the background gas.

50) Consider a two-dimensional ideal monatomic gas of N molecules of mass M at temperature T constrained to move only in the sy plane. The usual volume becomes in this case an area A, and the pressure p is the force per unit length (rather than the force per unit area). (a) Give an expression for f (v)dv, the total number of molecules with speeds between v and u + dv. (Assume that the classical limit is applicable in considering the behavior of these molecules). (b) Give the equation of state (relating pressure, temperature etc.). (c) Give the specific heats at constant area (two dimensional analogue (d) Derive a formula for the number of molecules striking unit length of the wall per unit time. Express your result in terms of N, A, T, M and any other necessary constants.

51) the schematic drawing below (Fig. 2.42) shows the experimental set up for the production of a well-collimated beam of sodium atoms for an atomic beam experiment. Sodium is present in the oven S, which is kept at the temperature T = 550 K. At this temperature the vapor pressure of sodium is p = 6 x low3 torr. The sodium atoms emerge through a slit in the wall of the oven. The hole is rectangular, with dimensions 10 mm x 0.1 mm. The collimator C has a hole of identical size and shape, and the sodium atoms which pass through C thus constitute the atomic beam under consideration. The atomic mass of sodium is 23. The distance d in the figure is 10 cm.

52) Consider air at room temperature moving through a pipe at a pressure low enough so that the mean free path is much longer than the diameter of the pipe. Estimate the net flux of molecules

in the steady state resulting from a given pressure gradient in the pipe. Use this result to calculate how long it will take to reduce the pressure in a tank of 100 litres volume from mm of Hg to lo-' mm of Hg, if it is connected to a perfect vacuum through a pipe one meter long and 10 cm in diameter. Assume that the outgassing from the walls of the tank and pipe can be neglected.

53) agas consists of a mixture of two types of molecules, having molecular masses MI and Mz grams, and number densities N1 and N2 molecules per cubic centimeter, respectively. The cross-section for collisions between the two different kinds of molecules is given by AlVl,l, where A is a constant, and V12 is the relative velocity of the pair. (a) Derive the average, over all pairs of dissimilar molecules, of the (b) How many collisions take place per cubic centimeter per second (UC, Berkeley) center-of-mass kinetic energy per pair. between dissimilar molecules?

54) a) What fraction of H_2 gas at sea level and T = 300 K has sufficient speed to escape from the earth's gravitational field? (You may assume an ideal gas. Leave your answer in integral form.) (b) Now imagine an H_2 molecule in the upper atmosphere with a speed equal to the earth's escape velocity. Assume that the remaining atmosphere above the molecule has thickness d = 100 km, and that the earth's entire atmosphere is isothermal and homogeneous with mean number density n = 2.5 x 1025/m3 (not a very realistic atmosphere). Using simple arguments, estimate the average time needed for the molecule to escape. Assume all collisions are elastic, and that the total atmospheric height is small compared with the earth's radius. Some useful numbers: Meartli = 6 x loz4 kg, Reartli = 6.4 x lo3 km.

55) What is the root-mean-square fluctuation in the number of photons of mode frequency w in a conducting rectangular cavity? Is it always smaller than the average number of photons in the mode?

56) Consider a system of non-interacting spins in an applied magnetic field H. Using S = k(1n Z + PE), where Z is the partition function, E is the energy, and /3 = l/kT, argue that the dependence of S on H and T is of the form S = f(H/T) where f(z) is some function that need not be determined. Show that if such a system is magnetized at constant T, then thermally isolated, and then demagnetized adiabatically, cooling will result. Why is this technique of adiabatic demagnetization used for refrigeration only at very low temperatures? How can we have T < 0 for this system? Can this give a means of achieving T = O?

57) An assembly of N fixed particles with spin - and magnetic moment /.LO is in a static uniform applied magnetic field. The spins interact with the applied field but are otherwise essentially free. (a) Express the energy of the system as a function of its total magnetic (b) Find the total magnetic moment and the energy, assuming that (c) Find the heat capacity and the entropy of the system under these (UC, Berkeley) moment and the applied field. the system is in thermal equilibrium at temperature T. same conditions.

58) Find the pressure, entropy, and specific heat at constant volume of an ideal Boltzmann gas of indistinguishable particles in the extreme relativistic limit, in which the energy of a particle is related to its momentum by E = cp. Express your answer as functions of the volume V, temperature T, and number of particle N.

59) consider a collection of N two-level systems in thermal equilibrium at a temperature T. Each system has only two states: a ground state of energy 0 and an excited state of energy E. Find each of the following quantities and make a sketch of the temperature dependence. (a) The probability that a given system will be found in the excited state. (b) The entropy of the entire collection.

60) a) Give a definition of the partition function z for a statistical system. (b) Find a relation between the heat capacity of a system and - a2 Inz aa2 ' 1 kT where P = -. (c) For a system with one

excited state at energy A above the ground state, find an expression for the heat capacity in terms of A. Sketch the dependence on temperature and discuss the limiting behavior for high and low temperatures.

61) (a) You are given a system of two identical particles which may occupy any of the three energy levels E, = n~, n = O,l, 2, . The lowest energy state, EO = 0, is doubly degenerate. The system is in thermal equilibrium at temperature T. For each of the following cases determine the partition function and the energy and carefully enumerate the configurations. 1) The particles obey Fermi statistics. 2) The particle obey Bose statistics. 3) The (now distinguishable) particles obey Boltzmann statistics. (b) Discuss the conditions under which Fermions or Bosons may be treated as Boltzmann particles.

62) Consider a system of two atoms, each having ony 3 quantum states of energies 0, E and 2s. The system is in contact with a heat reservoir at temperature T. Write down the partition function Z for the system if the particles obey (a) Classical statistics and are distinguishable. (b) Classical statistics and are indistinguishable. (c) Fermi-Dirac statistics. (d) Bose-Einstein statistics.

63) (a) Describe the third law of thermodynamics. (b) Explain the physical meaning of negative absolute temperature. Does it violate the third law? Why? (c) Suggest one example in which the negative temperature can actually be achieved. (d) Discuss why the negative temperature does not make sense in classical thermodynamics.

64) A system of N identical spinless bosons of mass rn is in a box of volume V = L3 at temperature T > 0. (a) Write a general expression for the number of particles, n(E), having an energy between s and E + ds in terms of their mass, the energy, the temperature, the chemical potential, the volume, and any other relevant quantities. (b) Show that in the limit that the average distance, d, between the particles is very large compared to their de Broglie wavelength (i.e., d >> A) the distribution becomes equal to that calculated using the classical (Boltzmann) distribution

function. (c) Calculate the 1st order difference in average energy between a system of N non-identical spinless particles and a system of N identical spinless bosons when d >> A. For both systems the cubical box has volume V = L3 and the particles have mass rn.

65) onsider a system of N non-interacting particles (N >> 1) in which the energy of each particle can assume two and only two distinct values, 0 and E (E > 0). Denote by no and nl the occupation numbers of the energy levels 0 and E, respectively. The fixed total energy' of the system is U. (a) Find the entropy of the system. (b) Find the temperature as a function of U. For what range of values (c) In which direction does heat flow when a system of negative temperature is brought into thermal contact with a system of positive temperature? Why?

66) he entropy of an ideal paramagnet in a magnetic field is given approximately by s=so-cu2, where U is the energy of the spin system and C is a constant with fixed mechanical parameters of the system. (a) Using the fundamental definition of the temperature, determine (b) Sketch a graph of U versus T for all values of T (-m < T < m). (c) Briefly tell what physical sense you can make of the negative temthe energy U of the spin system as a function of T. perature part of your result.

67) he response of polar substances (e.g., HC1, H20, etc) to applied electric fields can be described in terms of a classical model which attributes to each molecule a permanent electric dipole moment of magnitude p. (a) Write down a general expression for the average macroscopic polarization jj (dipole moment per unit volume) for a dilute system of n molecules per unit volume at temperature T in a uniform electric field E. (b) Calculate explicitly an approximate result for the average macroscopic polarization jj at high temperatures (KT > pE).

68) he molecule of a perfect gas consists of two atoms, of mass rn, rigidly separated by a distance d. The atoms of each molecule carry charges q and -q respectively, and the gas is placed in an electric field E. Find the mean polarization, and the specific heat per molecule, if quantum effects can be neglected. State the condition for this last assumption to be tru

69) A classical harmonic oscillator of mass m and spring constant k is known to have a total energy of E, but its starting time is completely unknown. Find the probability density function, p(x), where p(z)ds is the probability that the mass would be found in the interval dx at x.

70) Suppose there are two kinds of E. coli (bacteria), “red” ones and “green” ones. Each reproduces faithfully (no sex) by splitting into half, red-+red+red or green+green+green, with a reproduction time of 1 hour. Other than the markers “red” and “green”, there are no differences between them. A colony of 5,000 “red” and 5,000 “green” E. coli is allowed to eat and reproduce. In order to keep the colony size down, a predator is introduced which keeps the colony size at 10,000 by eating (at random) bacteria. (a) After a very long time, what is the probability distribution of the number of red bacteria?

71) (a) What are the reduced density matrices in position and momentum (b) Let us denote the reduced density matrix in momentum space by spaces? q5(pl, pz). Show that if q5 is diagonal, that is, d4Pl)PZ) = f(Pl)Lm

72) (a) Consider a large number of N localized particles in an external magnetic field H. Each particle has spin 1/2. Find the number of states accessible to the system as a function of M,, the z-component of the total spin of the system. Determine the value of M, for which the number of states is maximum. (b) Define the absolute zero of the thermodynamic temperature. Explain the meaning of negative absolute temperature, and give a concrete example to show how the negative absolute temperature can be reached.

73) (a) Give the definition of entropy in statistical physics. (b) Give a general argument to explain why and under what circumstances the entropy of an isolated system A will remain constant, or increase. For convenience you may assume that A can be divided into subsystems B and C which are in weak contact with each other, but which themselves remain in internal thermodynamic equilibrium

74) l meaning briefly but clearly. A two-level system of N = nl +n2 particles is distributed among two eigenstates 1 and 2 with eigenenergies El and E2 respectively. The system is in contact with a heat reservoir at temperature T. If a single quantum emission into the reservoir occurs, population changes n2 --+ n2 - 1 and n1 -+ nl+ 1 take place in the system. For nl >> 1 and n2 >> 1, obtain the expression for the entropy change of (a) the two level system, and of (b) the reservoir, and finally (c) from (a) and (b) derive the Boltzmann relation for the ratio nl/nz.

75) Consider a system composed of a very large number N of distinguishable atoms, non-moving and mutually non-interacting, each of which has only two (non-degenerate) energy levels: 0,s > 0. Let E/N be the mean energy per atom in the limit N -+ 00. (a) What is the maximum possible value of E/N if the system is not necessarily in thermodynamic equilibrium? What is the maximum attainable value of E/N if the system is in equilibrium (at positive temperature, of course)? (b) For thermodynamic equilibrium, compute the entropy per atom, (Prince ton) SIN, as a function of E/N.

76) a) Explain Boltzmann statistics, Fermi statistics and Bose statistics, especially about their differences. How are they related to the indistinguishability of identical particles? Statistical Phymca 175 (b) Give as physical a discussi6n as you can, on why the distinction between the above three types of statistics becomes unimportant in the limit of high temperature (how high is high?). Do not merely quote formulas. (c) In what temperature range will quantum statistics have to be applied to a collection of neutrons spread out in a two-dimensional plane with the number of neutrons per unit area being - 1012/cm2?

77) (a) State the basic differences in the fundamental assumptions under- (b) Make a rough plot of the energy distribution function at two different temperatures for a system of free particles governed by MB statistics and one governed by FD statistics. Indicate which curve corresponds to the higher temperature. (c) Explain briefly the discrepancy between experimental values of

the specific heat of a metal and the prediction of MB statistics. How did FD statistics overcome the difficulty?

78) State which statistics (classical Maxwell-Boltzmann; Fermi-Dirac; or Bose-Einstein) would be appropriate in these problems and explain why (semi-quantit atively) : (a) Density of He4 gas at room temperature and pressure. (b) Density of electrons in copper at room temperature. (c) Density of electrons and holes in semiconducting Ge at room tem- (VC, Berkeley) perature (Ge band-gap w 1 volt).

79) A long, thin (i.e., needle-shaped) dust grain floats in a box filled with gas at a constant temperature T. On average, is the angular momentum vector nearly parallel to or perpendicular to the long axis of the grain? Explain.

80) Find and make careful sketch of the probability density, p(E), for the energy E of a single atom in a classical non-interacting monatomic gas in thermal equilibrium.