

Reliability

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Reliability:

Qualitative & Quantitative Definition.

Qualitative Definition:

A reliability may be defined as the probability of a device to operate for a given period of time, without failure under a given operating conditions; for a given interval $[0,t]$.

Quantitative Definition:

Let T be a life length (or time to failure) of a component then the reliability is:

$$R(t) = P_r (T > t)$$

Properties of Reliability function:

1- Decreasing function.

2- $R(\infty) = 0$

3- $R(0) = 1$

$$* R(0) = 1$$

This means at the initial time $t=0$ the system (or component or device) is working perfectly.

$$* R(\infty) = 0$$

This means no device can work for ever.

$R(t)$ in terms of p.d.f $f(t)$ may be written as

$$R(t) = \int_t^{\infty} f(u) du$$

$R(t)$ in terms of c.d.f $F(t)$ may be expressed as

$$R(t) = 1 - F(t)$$

Unreliability function $Q(t)$

$$Q(t) = P_r(T \leq t)$$

$$R(t) + Q(t) = 1$$

Q/What it means to say $R(t)=0.90$?

It means that 10 percent will fail of the identical items under the same working condition in the interval $[0,t]$.

Reliability function or survival function:

The life time (T) is non negative r.v.

$$\begin{aligned} R(t) &= P_r(T > t) \\ &= P_r(\text{a component survives after age } t) \end{aligned}$$

$$R(t) = \int_t^{\infty} f(u) du$$

Distribution Function:

$$F(t) = P_r(T \leq t)$$

$$= P_r(\text{failure of a component time} \leq t)$$

$$F(t) = \int_0^t f(u) du$$

$$F(t) = Q(t) = 1 - R(t)$$

Probability of Failure (Failure Density function) :

The probability of failure may be defined as the unconditional probability of failure in the time interval (Δt) between (t) and ($t + \Delta t$).

$$f(t) = P_r(t < T \leq t + \Delta t)$$

Hazard function:

The hazard function may be defined as the conditional probability at a component to fail in the interval (Δt) , between $(t$ and $t + \Delta t)$ given that it has not failed until time (t) this may be expressed as:.

$$Z(t) = P(t < T \leq (t + \Delta t) / T > t)$$

$$Z(t) = \frac{P(t < T \leq (t + \Delta t))}{P(T > t)}$$

$$Z(t) = \frac{f(t)}{R(t)}$$

General Expression of Reliability

$$Z(t) = \frac{f(t)}{R(t)}$$

$$Z(t) = \frac{-R'(t)}{R(t)}$$

$$\int_0^t Z(s) ds = \int_0^t \frac{-R'(s)}{R(s)} ds$$

$$\int_0^t Z(s) ds = -\ln R(s) \Big|_0^t$$

$$\int_0^t Z(s) ds = -\ln R(t)$$

$$-\int_0^t Z(s) ds = \ln R(t)$$

$$R(t) = e^{-\int_0^t Z(s) ds}$$

Special case

if $Z(s)$ is Constant $\rightarrow \mathbf{Z}(s) = \lambda$

$$R(t) = e^{-\int_0^t Z(s)ds} = e^{-\int_0^t \lambda ds} = e^{-\lambda t}$$

$$f(t) = Z(t) * R(t)$$

$$f(t) = \lambda e^{-\lambda t}$$

Empirical Reliability:

N_O : Number of a component under the test.

$N_S(t)$: Number of survival component up to time t.

$N_F(t)$: Number of failure component up to time t.

$$\hat{P}_{Survival} = R(t) = \frac{N_S(t)}{N_O}$$

$$\hat{P}_{Failure} = Q(t) = \frac{N_F(t)}{N_O}$$

$$N_O = N_S(t) + N_F(t)$$

Mean time to failure (MTTF):

Is the mean time to the first failure (non repairable device).

$$MTTF = E(t) = \int_0^{\infty} tf(t)dt = \int_0^{\infty} R(t)dt$$

Mean time between failure(MTBF):

Is the expected time between two successive failure (repairable component).

$$\frac{1}{MTBF} = \sum_{i=1}^n \frac{1}{MTTF_i}$$

اكثر من فشل واحد

$$MTBF = MTTF = \frac{1}{\lambda}$$

إذا كانت فشل واحد

Theorem:

$$\mathbf{E(T)} = \int_0^{\infty} \mathbf{R(t)} dt$$

Proof: Left hand

$$\begin{aligned} E(t) &= \int_0^{\infty} t f(t) dt \\ &= - \int_0^{\infty} t \dot{R}(t) dt \end{aligned}$$

$$\begin{aligned} \text{Let: } u &= t & du &= dt \\ dv &= -\dot{R}(t) dt & v &= -R(t) \end{aligned}$$

$$E(t) = -t R(t) \Big|_0^{\infty} + \int_0^{\infty} R(t) dt$$

$$E(t) = -\infty R(\infty) - 0 R(0) + \int_0^{\infty} R(t) dt$$

$$\therefore E(t) = \int_0^{\infty} R(t) dt$$

Right hand

$$\int_0^{\infty} R(t) dt = \int_0^{\infty} \left[\int_t^{\infty} f(s) ds \right] dt$$

$$\text{Let: } u = \int_t^{\infty} f(s) ds \quad du = -f(t) dt$$

$$dv = dt \quad v = t$$

$$\int_0^{\infty} R(t) dt = t \int_t^{\infty} f(s) ds \Big|_0^{\infty} + \int_0^{\infty} tf(t) dt$$

$$= 0 + \int_0^{\infty} tf(t) dt$$

$$\therefore \int_0^{\infty} R(t) dt = \int_0^{\infty} tf(t) dt = E(t)$$

In special case

If $Z(t) = \lambda$

$$R(t) = e^{-\lambda t}$$

$$f(t) = \lambda e^{-\lambda t}$$

$$E(t) = \int_0^{\infty} R(t) dt$$

$$E(t) = \int_0^{\infty} e^{-\lambda t} dt = \frac{-1}{\lambda} e^{-\lambda t} \Big|_0^{\infty}$$
$$= \frac{-1}{\lambda} e^{-\lambda(\infty)} + \frac{1}{\lambda} e^{-\lambda(0)} = \frac{1}{\lambda}$$

$$E(t) = \frac{1}{\lambda} = MTTF$$

Theorem:

Let T be a time to failure of an item then the distribution of T is exponential, If and only if the failure rate is constant?

$$1 - T \sim \exp(\lambda) \quad \rightarrow \quad Z(t) \text{ is constant}$$

$$2 - Z(t) = \lambda \text{ constant} \quad \rightarrow \quad T \sim \exp(\lambda)$$

1 - T ~ exp(λ) **→ Z(t) is constant**

$$Z(t) = \frac{f(t)}{R(t)}$$

$$f(t) = \lambda e^{-\lambda t}$$

$$R(t) = P(T \geq t) = \int_t^{\infty} f(s) ds$$

$$= \int_t^{\infty} \lambda e^{-\lambda s} ds$$

$$= -e^{-\lambda s} \Big|_t^{\infty}$$

$$= e^{-\lambda t}$$

$$Z(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \quad \text{constant } Z(t)$$

2 – Z(t) = λ constant → T ~ exp(λ)

$$Z(t) = \lambda \quad \text{Constant}$$

$$R(t) = e^{-\int_0^t Z(s)ds} = e^{-\int_0^t \lambda ds} = e^{-\lambda t}$$

$$f(t) = Z(t) * R(t)$$

$$f(t) = \lambda e^{-\lambda t}$$

Example(1)

If $Z(t) = 3 * 10^{-5}$ find:

1- $R(t)$ for 100 hours.

2- What is the reliability equal MTTF?

Solution

$$Z(t) = \lambda = 3 * 10^{-5} \quad \text{constant}$$

$$1 - R(t) = e^{-\lambda t}$$

$$R(t = 100) = e^{-3*10^{-5}*100} = e^{-3*10^{-3}} = 0.99$$

$$2 - R(t = MTTF) = ?$$

$$MTTF = E(t) = \int_0^{\infty} R(t) dt$$

$$E(t) = \int_0^{\infty} e^{-\lambda t} dt = \frac{-1}{\lambda} e^{-\lambda t} \Big|_0^{\infty}$$
$$= \frac{-1}{\lambda} e^{-\lambda(\infty)} + \frac{1}{\lambda} e^{-\lambda(0)} = \frac{1}{\lambda}$$

$$MTTF = \frac{1}{\lambda}$$

$$R\left(t = \frac{1}{\lambda}\right) = e^{-\lambda t} = e^{-\lambda * \frac{1}{\lambda}} = e^{-1} = 0.37$$

Example(2)

Assume that (5000) items are put under the test, and if failure rate is constant $5 * 10^{-4}$ find:

1- Reliability for 500 hour.

2- $N_S(t)$ for 500 hours.

3- $N_f(t)$ for 500 hours.

Solution/

$$N_0 = 5000 \text{ items}$$

$$\lambda = Z(t) = 5 * 10^{-4}$$

$$1 - R(t) = e^{-\lambda t}$$

$$R(t = 500) = e^{-5 \cdot 10^{-4} \cdot 500}$$
$$= e^{-0.25} = 0.78$$

$$2 - N_S(t) = R(t) * N_0$$
$$= e^{-0.25} * 5000 = 3900$$

$$3 - N_F(t) = N_0 - N_S(t)$$
$$= 5000 - 3900 = 1100$$

Example(3):

If $\lambda=0.01$ parameter of exponential distribution and $R(t)=0.90$,
Find: t [the number of hours as a system operated]

Solution:

$$R(t) = e^{-\lambda t}$$

$$0.90 = e^{-0.01 * t}$$

$$\ln(0.90) = -0.01 * t$$

$$t = \frac{-\ln(0.90)}{0.01} = 10.5 \text{ hours}$$

Example(3):

If the Reliability for 100h equal to 0.99 find the failure rate:

Solution:

$$R(t = 100) = e^{-\lambda t} = 0.99$$

$$e^{-\lambda 100} = 0.99$$

$$\ln(0.99) = -\lambda 100$$

$$\lambda = \frac{-\ln(0.99)}{100} = 0.00003$$