Ministry of Higher Education and Scientific research

Department of Statistics and Informative

College of Administration and Economics

University of Salahaddin

Subject: Statistical Inference

Course Book – Fourth Stage (First Semester)

Lecturer's name: Lec. Zainab Abdulla Muhammad

Academic Year: 2023-2024

Ministry of Higher Education and Scientific research

Course Book for the Second Semester

1. Course name	Statistical Inference
2. Lecturer in charge	Lec. Zainab Abdulla Muhammad
3. Department/ College	Statistics/ Administration and Economics
4. Contact	e-mail: zainab.muhammad@su.edu.krd
5. Time (in hours) per week	(6) hours
6. Office hours	(6 hours) during the week
7. Course code	STE401
8. Teacher's academic profile	From 2006 until 2008 worked in Statistics Department - Salahaddin University. In 2011 I had my MSc. In Statistics from same University. From 2011 till now I am working as a Lecturer in Statistics Department- Salahaddin University.
9. Keywords	Probability Distributions of Random Variables. Transformations, Order Statistics, Point and Interval Estimation, Unbiasedness, Consistency and Sufficiency Estimator, Completeness, Uniqueness, Efficiency , Fisher Information , Maximum Likelihood Estimation . Minimum Variance Method Bayesian Estimation Method, Interval Estimation Neyman -Pearson Theorem, Testing of Statistical Hypotheses.

10. Course overview:

Statistical Inference is considered a topic in department of statistics, because at the beginning the student will get familiar with statistical distribution most of the researches are depending on these distributions for analysing data.

-Via statistics students will learn proving any rules and how they formed, we will make students learn them especially according to their distributions.

-How distribution of functions is found in different researches.

-How proved the properties of best estimators to discrete and continuous distributions.

-How is estimate the parameters of population by traditional method or by Bayesian method.

- How testing of Hypotheses for parameters of population.

The most important things that the students should keep the subject under control, we should consider this point.

1. The important of the subjects in mathematical statistics in the third stage, students should review the basic rules.
2. Memorizing or recognizing statistical rules which are (24) basic rules that we always consider them.
3. Students should make a connection between the previous subject and current one.
4. While displaying important points students should write them down because these notes are crucial for solving the questions.
5. Following up those questions that are left unsolved students should do their best to solve them.

11. Course objective:

1. Know what is Inference?

2. Know what is the estimation of parameter?

3. Understand hypothesis testing & the "types of errors" in decision making.

4. Know what the α-level means.

5. Learn how to use test statistics to examine hypothesis about population mean, proportion.

This course is divided into two parts. The first part deals with estimation (point estimation and confidence intervals), properties of an estimator, methods for finding estimators, and the second part deals with hypothesis tests.

Statistical inference is a formal process of using sample data to answer questions or to draw conclusions about a population (Estimating population parameters and testing hypotheses). Confidence intervals provide a method for using sample data to construct estimates of population characteristics, whereas hypothesis tests allow us to use sample data to decide between two competing claims, called hypotheses, about a population characteristic. Although confidence intervals and hypothesis tests are generally used for different purposes, they share a common goal of generalizing from a sample to a population.

12. Student's obligation

The attendance and completion of all tests, exams, assignments, reports.

13. Forms of teaching

Different forms of teaching will be use to reach the objectives of the course: data show PowerPoint presentations for the head titles and summary of conclusion, classification of material and any other illustrations. There will be classroom discussions and the lecture will give enough background to translate, solve, analyze, derive, and evaluate problems by using white board.

14. Assessment scheme

Grading: Grades will be assigned on a curve, using the following percentages: 5% Quizzes and the presence and absence of students, 35% Exams, 60% Final and Pass: 50%.

15. Student learning outcome:

The clarity of the basic objectives of subject for students, namely;

They Learned how to find distribution of random variables of functions by using transformation technique, and order statistics function (discrete or continuous) in univariate and bivariate cases, and how to apply it in real life. They knew the properties of best estimators for the population parameters, They knew how to estimates the population parameters.

Content article is appropriate to the requirements of the outside world and the labour market because it deals with all types of data in the outside world and the labour market.

The new things that the student learn through this article are: Learned how to test the hypotheses. Learned all the details about the common continuous and discrete distributions in the population and how to deal with it.

16. Course Reading List and References:

1. Introduction to Mathematical Statistics, 5th edition; By Robert V. Hogg and Craig,1995.

2. Introduction to Probability Theory and Statistical Inference, 3rd edition; By Harold J. Larson, 1982.

3. Statistical inference / George Casella, Roger L. Berger.-2nd edition 2002.

4. Principles of Statistical Inference, D.R. Cox, 2006.

5. An introduction to Probability and Mathematical Statistics, Rohatgi, V.K. , 1976.
6. Theory of Point Estimation, E.L. Lehmann George Casella 2nd edition 1998.
7. Statistical Distributions. Merran Evans, Nicholas Hastings, Brian Peacock, 3rd Edition, 2000.
8. Mathematical Statistics. Ferguson, T.S. 1968.
9. Statistical inference. Silvey 1973.
10. Bayesian Inference in Statistical Analysis. Box and Tiro 1973.
11. The Theory of Statistical Inference. Zacks, S.
12. Introduction to Probability and Statistical Inference. George Roussas 2003.
13. Probability and Mathematical Statistics. Prasanna Sahoo 2013.

17. The Topics: Contents	Lecturer's name Lec. Zainab Abdulla Six hours a week
Review subjects and laws of Mathematical Statistics, Statistical Distributions, Discrete and Continuous.	First week - 6 hrs 2023 /9 / 17
Distributions of Functions of Random Variables(Discrete & Continuous).	
Distribution of Order Statistics.	
Statistical Inference Concepts and Important Definitions about Statistical Inference.	
Estimation of Parameters (Point Estimation) (properties of an estimator) Unbiasedness.	
Biased Part & Unbiased in Limit Mean Square Error	
Consistency Estimator, The Score Function	
Sufficiency (method 1)	
Sufficiency (method 2 conditional)	
Sufficiency (method 3 factorization) Joint Sufficient Estimator	
The Exponential Class of Probability Density Functions	
Completeness	
First Midterm Exam	
Uniqueness Estimator (M.V.U.E)	
Efficiency (Relative Efficiency)	
Final Exam for the First Semester	

18. Practical Topics (If there is any)	
There isn't any Practical Topics	

19. Examinations:

Q1: let $X_1, X_2, \ldots, X_n$ be a random sample of size (n) rssn taken from C.U(0,1). let $Y_1 < Y_2 < \ldots < Y_n$ be the order statistics of this sample. **Find** the p.d.f. of Y_1, the j.p.d.f. of Y_1 and Y_n

Sol.: $X \sim C.U(0,1)$

$$\therefore\ f(x)=\begin{cases}\dfrac{1}{b-a} & , a\le x\le b\\ 0 & o.w\end{cases} = \begin{cases}1 & , 0\le x\le 1\\ 0 & o.w\end{cases}$$

$$F(x)=p(X\le x)=\int_0^x 1\,dx=\begin{cases}0 & , x\le 0\\ x & , 0<x<1\\ 1 & , x\ge 1\end{cases}$$

$$g(y_1)=n\ f(y_1)\ [1-F(y_1)]^{n-1} \quad , \quad a\le y_1\le b$$

$$when\ \ x=y_1 \ \Rightarrow\ \therefore\ f(y_1)=1\ ,\ F(y_1)=y_1$$

$$\therefore g(y_1)=n\,(1)\,[1-y_1]^{n-1} \quad , \quad 0\le y_1\le 1$$

$$=\begin{cases}n(1-y_1)^{n-1} & , 0\le y_1\le 1\\ 0 & o.w\end{cases}$$

$$g(y_i,y_j)=\frac{n!}{(i-1)!(j-i-1)!(n-j)!}\times$$

$$\times\ [F(y_i)]^{i-1}[F(y_j)-F(y_i)]^{j-i-1}[1-F(y_j)]^{n-j}f(y_i)\,f(y_j) \quad , \quad a<y_i<y_j<b$$

$$0 \qquad\qquad o.w$$

$$When,\ \ i=1\ ,\ j=n$$

$$\therefore g(y_1,y_n)=\frac{n!}{0!(n-2)!0!}(y_n-y_1)^{n-2}=\frac{n(n-1)(n-2)!}{(n-2)!}(y_n-y_1)^{n-2}$$

$$=n(n-1)\,(y_n-y_1)^{n-2} \qquad , 0<y_1<y_n<1$$

Q2: In a random sample of size (n). Is $T=\overline{X}$ unbiased estimator for $\phi(\theta)=\theta$ of Ber(θ).

Sol:

$$E(T)\overset{?}{=}\phi(\theta)=\theta$$

1) $X\sim Ber(\theta) \quad \Rightarrow \quad f(x)=\theta^x(1-\theta)^{1-\theta} \quad x=0,1 \qquad , E(X)=\theta$

$$E(T=\overline{X})=E(\overline{X})=E\left(\frac{\sum X_i}{n}\right)=\frac{n}{n}E(X)=\theta$$

$\therefore\ \overline{X}$ *is unbiased estimator for* θ.

2) $X\sim Poi(\theta) \quad \Rightarrow \quad f(x)=\dfrac{e^{-\theta}\ \theta^x}{x!} \quad x=0,1,2,.... \qquad , E(X)=\theta$

$$E(T=\overline{X})=E(\overline{X})=E\left(\frac{\sum X_i}{n}\right)=\frac{n}{n}E(X)=\theta$$

$\therefore\ \overline{X}$ *is unbiased estimator for* θ.

Q3: from Exp($1/\theta$). Is $\sum_{i=1}^{n}X_i$ sufficient estimator for θ? (by factorization theorem).

Sol:

$$f(x_1, x_2, \ldots, x_n; \theta) = g(\hat{\theta}; \theta) \cdot H(x)$$

$$X \sim Exp(1/\theta) \quad \Rightarrow f(x;\theta) = \theta e^{-\theta x} \quad , \because Xs \ \ are \ \ independent$$

$$f(x_1, x_2, \ldots, x_n; \theta) = \prod_{i=1}^{n} f(x_i;\theta) = f(x_1;\theta) \cdot f(x_2;\theta) \cdots f(x_n;\theta)$$

$$= \theta e^{-\theta x_1} \times \theta e^{-\theta x_2} \times \ldots\ldots \times \theta e^{-\theta x_n}$$

$$= \theta^n e^{-\theta \sum x_i} \times 1$$

$$= g(\hat{\theta} = \sum x_i; \theta) \cdot H(x)$$

$$\therefore \hat{\theta} = \sum X_i \ \ is \ \ suff \ \ est \ \ for \ \ \theta.$$

Q4: Let X be a random variable from Poisson dist^n. Show that the family of X is complete.

Sol: $X \sim Poi(\theta)$

$$f(x;\theta) = \frac{e^{-\theta}\theta^x}{x!} \quad , x = 0,1,\ldots\ldots.$$

Let $u(x)$ *be a continuous fun of* X*. then;*

$$E(u(X)) = 0$$

$$E(u(X)) = \sum_{x=0}^{\infty} u(x) \ f(x;\theta)$$

$$= \sum_{x=0}^{\infty} u(x) \frac{e^{-\theta}\theta^x}{x!} = 0$$

$$\Rightarrow e^{-\theta} \sum_{x=0}^{\infty} u(x) \frac{\theta^x}{x!} = 0 \quad \} \div e^{-\theta}$$

$$\Rightarrow \sum_{x=0}^{\infty} u(x) \frac{\theta^x}{x!} = 0$$

$$\Rightarrow u(0) + u(1)\theta + u(2)\frac{\theta^2}{2!} + u(3)\frac{\theta^3}{3!} + \ldots\ldots = 0$$

$$\because \theta > 0$$

$$\therefore u(0) = u(1) = u(2) = u(3) = \ldots\ldots = 0 \qquad \Rightarrow u(x) = 0 \ \ \forall \ x$$

$$\therefore f(x;\theta) \ \ of \ \ Poisson \ \ dist \ \ is \ \ complete$$

20. Extra notes:

There isn't any extra notes or comments

21. Peer review پێداچوونەوەی هاوەڵ