Q1/ Show that by Mathematical **Induction** $3^n - 1$ is divisible by **2,** where **n** is any **positive Integer** number. (5M)

Q2/ let $(x, -y)\,(3, -4) = (3, -29)$ find the value of **x** and **y**. (5M)

Q3/ if $a + bi = \frac{2-i}{1-2i}$ where $a \ and \ b \in R$, evaluate $a^2 + b^2$. (5M)

Q1/ prove the statements by **Mathematical Induction**, where n is any positive Integer number. (7+7M)

1) $1(2) + 2(3) + 3(4) + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

2) $n^2 + n$ is even number.

Q2/A/ simplify the following:- (6+6M)

1) $\frac{2}{8+3i} + \frac{1}{8-3i}$ 2) $(\overline{4 - 3i})^2$

B/ find the value of $k_1 \ and \ k_2$ in the following:- (6M)

$(2k_1 - 3k_2) + 6i(k_1 - k_2) = 2 - i(2k_1 - k_2 + 3)$

Q3/ A/ use **De Moivre's** Theorem to simplify the expression $(2\sqrt{3} - 2i)^{50}$. (7M)

B/ convert the complex number $z = -2 - 2i$ to the **Euler** form. (5M)

Q4/A/ using **De Moivre's** Theorem to prove $\sin(2x) = 2\sin(x)\cos(x)$. (8M)

B/ find all **roots** of the equation $z^2 = 5\sqrt{3} + 5i$ (8M)

**Worked Examples**

Plot the following numbers in the complex plane and find $|z|$, $Arg(z)$ and $arg(z)$ for:

(a) $z = 1 + i$

(b) $z = 1 - i$

(c) $z = -\sqrt{3} - i$

(d) $z = -1 + \sqrt{3}i$

(e) $z = i$

(f) $z = 2$

(g) $z = -\pi$

## Check your understanding

Find $Arg(z)$ and $arg(z)$ if:

(i) $\sqrt{3} + i$

(ii) $1 - \sqrt{3}i$

(iii) $8i$

(iv) $-1 + i$

(v) $-3 - 3i$

**Worked Examples 1.10.1.**

Write the following complex numbers in the modulus-argument form.

(a) $z = 1 + i$

(b) $z = 1 - i$

(c) $z = -\sqrt{3} - i$

(d) $z = -1 + \sqrt{3}i$

**Eg** Convert the following complex numbers from one form to the other.

1. $z = 3i$;
2. $z = 1$;
3. $z = 1 + i\sqrt{3}$;
4. $z = -2 - 2i$;
5. $z = e^{-i\frac{\pi}{6}}$;
6. $z = 5e^{i\frac{\pi}{4}}$;
7. $z = -5e^{-i\frac{\pi}{3}}$.

**EXAMPLE** ***Finding the Modulus of a Complex Number***

For $z = 2 + 3i$ and $w = 6 - i$, determine the following.

(a) $|z|$ (b) $|w|$ (c) $|zw|$

**Solution** (a) $|z| = \sqrt{2^2 + 3^2} = \sqrt{13}$

(b) $|w| = \sqrt{6^2 + (-1)^2} = \sqrt{37}$

(c) Since $zw = (2 + 3i)(6 - i) = 15 + 16i$, we have

$$|zw| = \sqrt{15^2 + 16^2} = \sqrt{481}.$$

## *Properties of modulus of a complex number*

1. $|z| = 0 \Leftrightarrow z = 0$ i.e., $\text{Re}(z) = 0$ and $\text{Im}(z) = 0$

2. $|z| = |\bar{z}| = |-z|$

3. $-|z| \le \text{Re}(z) \le |z|$ and $-|z| \le \text{Im}(z) \le |z|$

4. $z\bar{z} = |z|^2$, $|z^2| = |\bar{z}|^2$

5. $|z_1 z_2| = |z_1| \cdot |z_2|$, $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ $(z_2 \ne 0)$

6. $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1 \bar{z}_2)$

7. $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\,\text{Re}\,(z_1 \bar{z}_2)$

8. $|z_1 + z_2| \leq |z_1| + |z_2|$

9. $|z_1 - z_2| \geq |z_1| - |z_2|$

10. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

In particular:

$$|z_1 - z_2|^2 + |z_1 + z_2|^2 = 2\,(|z_1|^2 + |z_2|^2)$$

11. As stated earlier multiplicative inverse (reciprocal) of a complex number $z = a + ib\ (\neq 0)$ is

$$\frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\bar{z}}{|z|^2}$$

**9.** If $a+bi=\frac{9-7i}{2-3i}$, find the value of $a$ and the value of $b$, $a, b \in \mathbf{R}$.

**10.** If $p+qi=\frac{2-i}{1-2i}$, $p, q \in \mathbf{R}$, evaluate $p^2+q^2$.

**11.** Given that $(4+3i)z=1+7i$, express the complex number $z$ in the form $a+bi$.

**12.** **(i)** Express $(1-2i)^2$ in the form $a+bi$.

**(ii)** Hence, find the real part of $\frac{1}{(1-2i)^2}$.

Express each of the following in the form $a+bi$, where $a, b \in \mathbf{R}$ and $i^2=-1$:

**1.** $\frac{3+4i}{2+i}$ **2.** $\frac{7+4i}{2-i}$ **3.** $\frac{1+5i}{3+2i}$ **4.** $\frac{7-17i}{5-i}$

**5.** $\frac{1}{1-i}$ **6.** $\frac{2+i}{1+2i}$ **7.** $\frac{2-i}{3+2i}$ **8.** $\frac{3+4i}{1-i}$

## Exercise

Q.1: Write each ordered pair (complex number) in the form: $a + bi$.

(i) $(2, 6)$ (ii) $(5, -2)$

(iii) $(-7, -3)$ (iv) $(4, 0)$

Q.2: Write each complex number as an ordered pair.

(i) $(2 + 3i)$ (ii) $(-3 + i)$ (iii) $(4i)$ (iv) $(0)$

Q.3: Find the value of x and y in each of the following:

(i) $x + 3i + 3 = 5 + yi$ (ii) $x + 2yi = ix + y + 1$

(iii) $(x, y)(1, 2) = (-1, 8)$ (iv) $(x, -y)(3, -4) = (3, -29)$

(v) $(2x - 3y) + i(x - y)6 = 2 - i(2x - y + 3)$

In Exercises 5–18, prove the statements by induction.

5. $3 + 5 + \cdots + (2n + 1) = n(n + 2)$

6. $2 + 6 + 10 + \cdots + (4n - 2) = 2n^2$

7. $1 + 4 + 7 + \cdots + (3n - 2) = \dfrac{n(3n - 1)}{2}$

8. $5 + 4 + 3 + \cdots + (6 - n) = \dfrac{1}{2}n(11 - n)$

9. $7 + 5 + 3 + \cdots + (9 - 2n) = -n^2 + 8n$

10. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n + 1)(2n + 1)}{6}$

11. $1^3 + 2^3 + \cdots + n^3 = \dfrac{n^2(n + 1)^2}{4}$

12. $\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \dfrac{1}{2 \cdot 3} + \cdots + \dfrac{1}{n(n + 1)} = \dfrac{n}{n + 1}$

13. $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n + 1) = \dfrac{n(n + 1)(n + 2)}{3}$.