For each polynomial function find the values indicated.

a $P(x) = 3x^2 - 7x + 1$ find $P(0)$, $P(1)$, $P(-2)$

b $Q(x) = 25x - 27$ find $Q(-1)$, $Q(1)$, $Q(0)$

c $R(x) = 3x^6$ find $R(2)$, $R(-3)$, $R(0)$

e $(x + 5)(x - 2)(3x + 1)(4x - 3)$

f $(x - 2)(x - 5)(2x + 1)(3x - 1)$

Find the sum $P(x) + Q(x)$ and the difference $P(x) - Q(x)$ of the polynomials given.

i $P(x) = 3x^2 - 5x + 2$, $Q(x) = x^2 + 7x - 3$

ii $P(x) = x^3 - 2x^2 + 3x + 7$, $Q(x) = 2x^3 + 3x^2 - 5x + 2$

iii $P(x) = 2x^3 + 9x^2 + 8x + 1$, $Q(x) = 6x^2 - 5x - 3$

iv $P(x) = 3x^4 + 2x^3 - 3x^2 + 4x - 2$, $Q(x) = 2x^4 + 3x^3 - 7x^2 + 5x - 11$

v $P(x) = x^4 + 9x^3 + 7x^2 - 4x + 9$, $Q(x) = 4x^3 - 8x^2 - 15$

For each of the following, show by division that the first polynomial is a factor of the second polynomial.

a $x + 2$, $x^3 + 5x^2 + 5x - 2$

b $2x - 1$, $2x^3 - x^2 + 2x - 1$

c $x + 1$, $x^3 + 3x^2 + 3x + 1$

d $3x + 4$, $3x^3 + 10x + 11x + 4$

e $x - 2$, $x^4 + x^3 - 8x^2 + 5x - 2$

f $x + 2$, $x^4 - x^3 - 9x^2 + 3x + 18$

Express each polynomial $P(x)$ as the product of three linear factors given that $h(x)$ is one of these factors.

a $P(x) = x^3 + 4x^2 + x - 6$, $h(x) = x - 1$

b $P(x) = 2x^3 - 5x^2 - x + 6$, $h(x) = 2x - 3$

c $P(x) = 2x^3 - 15x^2 + 22x + 15$, $h(x) = x - 5$

d $P(x) = 4x^3 + 12x^2 - x - 3$, $h(x) = x + 3$

e $P(x) = 8x^3 + 36x^2 + 54x + 27$, $h(x) = 2x + 3$

f $P(x) = 8x^3 - 12x^2 + 6x - 1$, $h(x) = 2x - 1$

Find the quotient and remainder for each of the following divisions.

a $(3x^3 - 2x^2 - 27x - 18) \div (3x - 2)$

b $(-6x^3 + x^2 + 4x + 3) \div (2x + 1)$

c $(5x^3 + 7x - 4) \div (x - 3)$

d $(x^3 - 1) \div (x + 1)$

e $(6x^3 - 5x^2 - 2) \div (2x - 1)$

f $(x^4 - a^4) \div (x - a)$

g $(4x^3 - 7x^2 + 2x - 3) \div (x^2 + 2x - 1)$

h $(2x^3 + 5x^2 + 11x - 1) \div (x^2 - x + 3)$

Use the remainder theorem to find the remainder when the first polynomial is divided by the second.

a $2x + 7, x + 1$

b $3x^2 + 12x - 28, x - 1$

c $5x^2 - 11x - 31, x - 4$

d $x^3 - 4x^2 + 3x + 1, x + 1$

e $-x^3 + 2x^2 - 7x - 13, x + 5$

f $2x^3 + 7x^2 - 10x - 5, x - 3$

g $-4x^3 - x^2 + x + 1, x + 2$

h $x^4 + x^3 - 2x^2 - 4x + 3, x + 2$

i $3x^4 - 5x^3 - 6x^2 + 8x - 6, x - 2$

j $2x^4 - 25x^3 + 71x^2 - 56x + 35, x$

Use the factor theorem to find all the linear factors of the following polynomials.

a $x^3 + 3x^2 - 6x - 8$

b $x^3 - 7x + 6$

c $x^3 + 3x^2 - 16x + 12$

d $x^3 + 6x^2 + 12x + 8$

e $x^3 + 6x^2 - x - 6$

f $x^3 + 3x^2 - 10x - 24$

g $x^3 - 7x^2 + 36$

h $x^4 + x^3 - 3x^2 - 4x - 4$

i $x^4 + 2x^3 - 3x^2 - 4x + 4$

j $2x^3 - 13x^2 - 13x + 42$

a If $x - 3$ is a factor of $6x^3 + kx^2 + 2x + 3$, find the value of k.

b If $P(x) = x^3 + ax^2 + bx - 6$ is divisible by $(x + 1)$ and $(x - 2)$, find the values of a and b.

c $(x + 1)$ is a factor of $f(x) = x^3 + mx^2 + nx - 3$, and when $f(x)$ is divided by $(x + 3)$ the remainder is -24. Find m and n.

Exercise Set

In Exercises 1–4, use Descartes' Rule of Signs to determine the maximum number of positive and negative zeros.

1. $P(x) = 6x^4 + 5x^3 - 14x^2 + x + 2$
2. $P(x) = 9x^4 - 9x^3 - 19x^2 + x + 2$
3. $P(x) = x^5 + 2x^4 - x - 2$
4. $P(x) = x^5 - 2x^4 - 9x^3 + 8x^2 - 22x + 24$

Use the Rational Root Theorem to list all possible rational roots for each equation.

1. $x^3 - 4x + 1 = 0$ **2.** $x^3 + 2x - 9 = 0$ **3.** $2x^3 - 5x + 4 = 0$

4. $3x^3 + 9x - 6 = 0$ **5.** $4x^3 + 2x - 12 = 0$ **6.** $6x^3 + 2x - 18 = 0$

7. $7x^3 - x^2 + 4x + 10 = 0$ **8.** $8x^3 + 2x^2 - 5x + 1 = 0$ **9.** $10x^3 - 7x^2 + x - 10 = 0$

1) $\boldsymbol{find\ AB}$

$$\begin{bmatrix} 2 & -1 \\ 4 & 3 \\ -3 & 0 \end{bmatrix} \quad and\ B = \begin{bmatrix} 7 & -2 & -3 \\ 5 & 1 & 4 \end{bmatrix}$$

Given the following matrices, please solve the questions below and if you can't solve the problem, explain why:

A = $\begin{bmatrix} 3 & 1 & 5 \\ 6 & 2 & 0 \end{bmatrix}$ B = $\begin{bmatrix} 6 \\ 4 \\ -1 \end{bmatrix}$ C = $\begin{bmatrix} 2 & 4 \\ 3 & 6 \\ -1 & 2 \end{bmatrix}$ D = $\begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$ E = $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$

F = $\begin{bmatrix} 2 & 1 & 3 \\ 5 & 7 & -2 \end{bmatrix}$

1) A + F 2) E – D 3) C + B 4) C(D) 5) A(F) 6) C^T 7) F^T(E)

Exercise

Calculate the determinant of the following matrices:

$$a)\begin{pmatrix} 1 & 3 & 2 \\ 4 & 1 & 3 \\ 2 & 2 & 0 \end{pmatrix} \qquad b)\begin{pmatrix} 1 & 0 & 2 \\ 1 & 3 & 4 \\ 0 & 6 & 0 \end{pmatrix}$$

$$c)\begin{pmatrix} 3 & -2 & 4 \\ 2 & -4 & 5 \\ 1 & 8 & 2 \end{pmatrix} \qquad d)\begin{pmatrix} 8 & -1 & 9 \\ 3 & 1 & 8 \\ 11 & 0 & 17 \end{pmatrix}$$

Solutions : a) 24 b) -12 c) -66 d) 0

$$let\ A = \begin{pmatrix} 8 & -2 & 7 \\ -2 & -9 & 3 \\ 7 & 3 & 5 \end{pmatrix}\ ,\ find\ A^T\ and\ show\ that\ A^2\ is\ symmetric\ matrix$$

Exercise

Q.1 Which of the following matrices are singular or non-singular.

(i) $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & 5 \\ -4 & 2 & 6 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ 3 & 3 & -6 \end{bmatrix}$

Q.2 Which of the following matrices are symmetric and skew-symmetric

(i) $\begin{bmatrix} 2 & 6 & 7 \\ 6 & -2 & 3 \\ 7 & 3 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$

Q.3 Find K such that the following matrices are singular

(i) $\begin{vmatrix} K & 6 \\ 4 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & K \\ -4 & 2 & 6 \end{vmatrix}$ (iii) $\begin{vmatrix} 1 & 1 & -2 \\ 3 & -1 & 1 \\ k & 3 & -6 \end{vmatrix}$

Questions 22–28 are about the Gauss-Jordan method for calculating A^{-1}.

22 Change I into A^{-1} as you reduce A to I (by row operations):

$$[\,A \quad I\,] = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{bmatrix} \quad \text{and} \quad [\,A \quad I\,] = \begin{bmatrix} 1 & 4 & 1 & 0 \\ 3 & 9 & 0 & 1 \end{bmatrix}$$

23 Follow the 3 by 3 text example but with plus signs in A. Eliminate above and below the pivots to reduce $[\,A \quad I\,]$ to $[\,I \quad A^{-1}\,]$:

$$[\,A \quad I\,] = \begin{bmatrix} 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}.$$

1! Find the numbers a and b

$$\begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{bmatrix}.$$

Exercises

1. Solve the following using Cramer's rule:

(a) $\begin{array}{rcrcl} 2x & - & 3y & = & 1 \\ 4x & + & 4y & = & 2 \end{array}$ (b) $\begin{array}{rcrcl} 2x & - & 5y & = & 2 \\ -4x & + & 10y & = & 1 \end{array}$ (c) $\begin{array}{rcrcl} 6x & - & y & = & 0 \\ 2x & - & 4y & = & 1 \end{array}$

2. Using Cramer's rule obtain the solutions to the following sets of equations:

(a) $\begin{array}{rcrcrcl} 2x_1 & + & x_2 & - & x_3 & = & 0 \\ x_1 & & & + & x_3 & = & 4 \\ x_1 & + & x_2 & + & x_3 & = & 0 \end{array}$ (b) $\begin{array}{rcrcrcl} x_1 & - & x_2 & + & x_3 & = & 1 \\ -x_1 & & & + & x_3 & = & 1 \\ x_1 & + & x_2 & - & x_3 & = & 0 \end{array}$