Mathematics Department fourth stage

Subject: Statistical Inference

Q1: A/ let $\boldsymbol{x}$ has the probability mass function (p.m.f.) (7 Marks)

$$\boldsymbol{f(x) = \frac{\theta^x e^{-\theta}}{x!}} \quad \boldsymbol{, x = 0, 1, 2, \dots}$$; find the p.m.f. of $\boldsymbol{y = 3x + 1}$.

B/ let $\boldsymbol{x}$ and $\boldsymbol{y}$ be two independent random variables that have probability density function (p.d.f.) $\boldsymbol{f(x) = \frac{1}{2} e^{\frac{-1}{2}x}}$, $\boldsymbol{x > 0}$, and $\boldsymbol{f(y) = 4e^{-4y}}$, $\boldsymbol{y > 0}$, find joint probability distribution function of $\boldsymbol{u}$ and $\boldsymbol{v}$, where $\boldsymbol{u = \frac{x}{y}}$, $\boldsymbol{and\ v = x}$. (10 Marks)

Q2: let $\boldsymbol{x_1, x_2, \dots, x_4}$ be a random sample of size 4 with p.d.f. (8+7 Marks)

$$\boldsymbol{f(x) = 12\, x\, (1 - x)^2\ ,\ 0 < x < 1}$$

find (1) p.d.f. of largest value (2) $\boldsymbol{g(y_1, y_3)}$.

Q3: let $\boldsymbol{x_1, x_2, \dots, x_n}$ be a random sample of size n with Normal distribution $\boldsymbol{N(\theta, 1)}$,

(1) find Maximum Likelihood Estimation (MLE) for θ.

(2) which estimator $\boldsymbol{\hat{\theta}_1\ or\ \hat{\theta}_2}$ is better? Using mean square error (MSE)
where $\boldsymbol{\hat{\theta}_1 = \frac{x_1 + x_2}{2}}$, $\boldsymbol{\hat{\theta}_2 = \frac{x_1 + 3x_2 + 2x_3}{6}}$.

(3) Is $\boldsymbol{\overline{X}}$ consistent estimator for $\boldsymbol{\theta}$? (8+10+10 Marks)

Q4/ let x and y be two random variables with j.p.d.f. given by the following table, and let $u = |x|$, $v = y^2$ find f(u,v). (10M)

y \ x	-1	0	1
-2	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{6}$
1	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{6}$
2	$\frac{1}{12}$	0	$\frac{1}{12}$

Q5: let $\boldsymbol{y_1 < y_2 < y_3}$ be order statistics of size 3 with p.d.f. (8+8+8 Marks)

$$\boldsymbol{f(x) = \theta e^{-\theta x}\ ,\ x > 0}$$

find (1) $\boldsymbol{g(y_2)}$

(2) $\boldsymbol{g(y_1, y_2, y_3)}$.

(3) Is $\boldsymbol{2y_2}$ **unbiased** estimator for $\boldsymbol{\theta}$?

Q6: let $\boldsymbol{x_1, x_2, \dots, x_n}$ be a random sample of size n from **Bernoulli** distribution with parameter $\boldsymbol{\theta}$.

(1) find minimum variance unbiased estimator (**M.V.U.E**) for $\boldsymbol{\theta}$.

(2) Is $\boldsymbol{\overline{X} + \frac{2}{n}}$ consistent estimator for $\boldsymbol{\theta}$? (10+10 Marks)

Q7/ let $x_1, x_2, \dots, x_n$ be r.s.s.n. From Bernoulli distribution with parameter θ, find an estimator for θ using

1) Method of moment (10M)
2) Maximum Likelihood Estimation. (10M)

Q8/ let $y_1 < y_2 < y_3$ be the order statistics of random sample of size 3 from a distribution with p.d.f. $f(x) = e^{-x},\ x > 0,$ find $E(y_2)$. (10M)

Q9: A/ if $\boldsymbol{f(x) = \begin{cases} \frac{1}{\theta} & 0 < x < \theta \\ 0 & otherwise \end{cases}}$ (8 Marks)

Find probability density function (p.d.f.) of $\boldsymbol{y = 2x + 1}$.

B/ let $\boldsymbol{f(x) = \frac{2^x e^{-2}}{x!} \quad x = 0, 1, 2, \ldots}$ and $\boldsymbol{f(y) = \frac{3^y e^{-3}}{y!} \quad y = 0, 1, 2, \ldots}$;
where x and y are independent random variables, let $\boldsymbol{u = x + y}$ and $\boldsymbol{v = x}$, find $\boldsymbol{h(u, v)}$.

(8 Marks)